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THE CROSS-SECTION PAPER AS A MATHEMATICAL INSTRUMENT.*

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The following note[1] is addressed to teachers and prospective teachers of mathematics in elementary and secondary schools and colleges, who have come to recognize as fundamental the problem of closer *correlation of arithmetic, algebra, and geometry with one another and with the various domains of application*, or the problem of *unification of elementary pure and applied mathematics*. Pure mathematics is, as it were, a language for the convenient expression and investigation of relations the most diverse in ordinary life and in nature. The principles of the language are not arbitrary, but are imposed by the phenomena demanding convenient expression. The problem is solved when, and only when, we put our pupils into such a physical and intellectual environment that they learn to see and to think the mathematics for and of themselves.

In this note I wish to suggest possibilities, which may have escaped attention, in the systematic use of cross-section paper[2] as a unifying

[1]Read in part March 5, 1906, before a joint meeting of the Junior Mathematical Club and the Mathematical Club of the University High School of the University of Chicago.

[2]In the few diagrams to be given here only the square-ruled paper appears. I advise the use of the various styles of ruling—into squares, into rectangles, into parallelograms, into triangles, and with concentric circles and diverging radii—obtainable, for instance, from the Atlas School Supply Co., Chicago. The interaction of the various papers is especially important.

*The University of Chicago Press has done SCHOOL SCIENCE AND MATHEMATICS the kindness of placing at its disposal, for a small charge, the electrotypes for Professor Moore's paper as it appeared in the *School Review* for May. This article is printed from these excellent types. The editors desire here to acknowledge their obligation for the privilege to use them.

element in mathematics. I know of no medium serving to bring together so closely and so easily the three phases or dialects of pure mathematics—*number, form, formula*—and to lead so directly to the concept of *functionality*—a concept which since the seventeenth century has dominated advanced mathematics and the sciences; a concept which in the twentieth century, according to the auspices, will play a fundamental rôle in the reorganization of elementary mathematical education. ***Functionality is the relation or (mathematical) law of connection between two or more quantities or numbers subject to simultaneous and interdependent continuous variation;*** like, for instance, the relation between the simultaneous ages of a father and his son; or that between the length of the perimeter and the area of the surface of a square surrounded by a platoon of men preserving square formation and marching each directly away from a fixed flagpole; or that between the simultaneous temperature, pressure, and density of a gas under conditions varying in time or in space.

Students will gain a more easy and perfect mastery of mathematics, and their work will be full of richer direct and indirect value for them, when the primary emphasis is laid on the recognition, the depiction, and the closer study of functional relations between variable quantities. The variable quantity is the general quantity. I recognize the difficulties inherent in the notions of general and signed quantities and numbers, and in the notion of the depiction of a quantity of one kind to scale by a quantity of another kind. But in elementary teaching these difficulties should be met not primarily by the analytical processes of logic resulting in the separation of mathematics into arithmetic, algebra, geometry, only loosely related and not readily available for application. They should be met by the systematic union[1] of the three phases of mathematics in connection with problems

[1]The best modern textbooks on arithmetic show in a gratifying way that for the elementary school the problem of unification is under way to solution. The authors develop in organic relation to problems of real life, not only number work, but also to a considerable degree literal arithmetic, observational geometry, geometrical drawing, drawing to scale, plotting, and algebra.

Further, it is known that gratifying progress is making in various quarters toward the solution of the problem for the secondary school. Cf. the columns of the *School Review* and of *School Science and Mathematics*, Chicago, and J. W. A. Young's book, *The Teaching of Mathematics*, to be published soon by Longmans, Green & Co., for information as to the current movements in this country and in England, France, and Germany.

of practical and scientific nature. So approached, these fundamental conceptions are not really so difficult. Every boy loves trains, and most boys have traveled. With the cross-section paper in commonplace school use, the boy of nine or ten will readily understand and create diagrams of train motion, and will enjoy making the limited express overtake the slow freight at a certain time and place.

In this note, confining attention to the simplest algebraic and geometric aspects of functionality, I study the cross-section paper in connection with the questions of *double-entry tables* and *graphical computation*, and I hope to suggest in some measure the very central rôle which the cross-section paper as a mathematical instrument should be made to play in the unification of the elementary mathematical disciplines. By maximizing the function of the cross-section paper we secure, to speak only of pure mathematics, intense reaction between geometry and algebra. Geometry and algebra may certainly be developed independently, each with its relations to arithmetic, and no one doubts their high educational and scientific value as so developed. But this value is indeed small compared with the value to be obtained by developing them together in continuous reaction, thus releasing, as it were, abundant stores of sub-atomic energy.

The reader is requested to have at hand pencil, straight edge, and cross-section paper, and to duplicate the constructions shown in the figures, reserving for a second reading the more general parts of the text.

Double-Entry Tables—Numerical and Graphical

1. The fundamental operations of arithmetic and algebra involve two numbers, and the results of addition, etc., should be exhibited in double-entry tables, as indicated below, where for brevity the subtraction table is omitted and the given numbers X, Y are taken as positive integers at most 5. These tables, exhibiting the sum S, etc.,

$$X+Y=S\,,\quad XY=P\,,\quad \frac{Y}{X}=Q\,,$$

of the two numbers X, Y are to be thought of as inscribed on square-ruled paper, each entry S or P or Q being made at the corresponding point (X, Y) of the paper. Make such tables on large sheets with entries for X and Y varying, for instance, by fifths from -10 to $+10$,

and then see clearly how $S=X+Y$, $D=Y-X$, $P=XY$, $Q=Y/X$, or any function[1] F of X and Y as a function of position of the point (X, Y) varies (in general) continuously as the point (X, Y) moves continuously.

2. Evidently the numerical double-entry tables (I, II, III) may conveniently be replaced by the graphical tables[2] (Figs. 1, 2, 3);

Y	X=1	2	3	4	5
5	6	7	8	9	10
4	5	6	7	8	9
3	4	5	6	7	8
2	3	4	5	6	7
1	2	3	4	5	6

I. ADDITION TABLE: $X+Y=S$

Y	X=1	2	3	4	5
5	5	10	15	20	25
4	4	8	12	16	20
3	3	6	9	12	15
2	2	4	6	8	10
1	1	2	3	4	5

II. MULTIPLICATION TABLE: $XY=P$

Y	X=1	2	3	4	5
5	5	$2\frac{1}{2}$	$1\frac{2}{3}$	$1\frac{1}{4}$	1
4	4	2	$1\frac{1}{3}$	1	$\frac{4}{5}$
3	3	$1\frac{1}{2}$	1	$\frac{3}{4}$	$\frac{3}{5}$
2	2	1	$\frac{2}{3}$	$\frac{2}{4}$	$\frac{2}{5}$
1	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$

III. DIVISION TABLE: $\frac{Y}{X}=Q$

Y	X=1	2	3	4	5
5	5.00	2.50	1.67	1.25	1.00
4	4.00	2.00	1.33	1.00	0.80
3	3.00	1.50	1.00	0.75	0.60
2	2.00	1.00	0.67	0.50	0.40
1	1.00	0.50	0.33	0.25	0.20

III. DIVISION TABLE: $\frac{Y}{X}=Q$

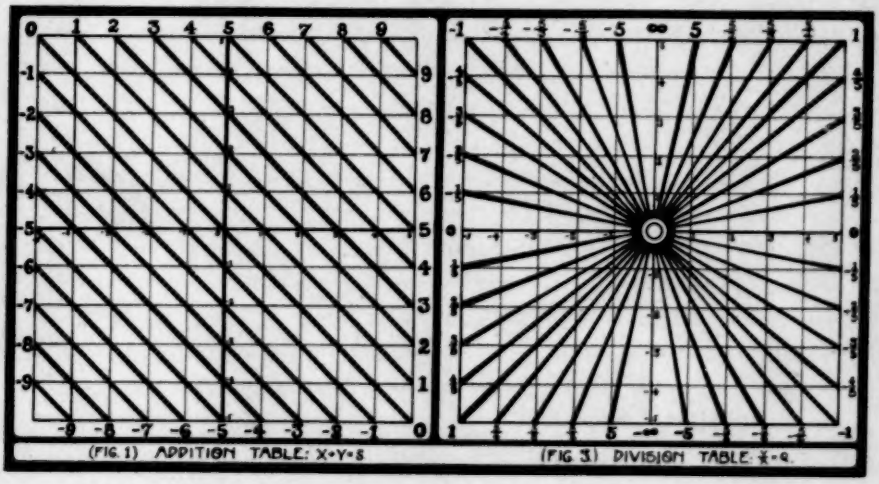
instead of making entries of the numerical values of S, etc., at individual points (X, Y) of the table, all the points (X, Y) with a certain

[1] A function F of two variable numbers X and Y is a variable number whose particular value for given values of X and Y is given or determinable in accordance with some table or formula or construction.

[2] The figures of this note are from drawings by Mr. J. Y. Lee, one of my pupils, to whom I express my gratitude.

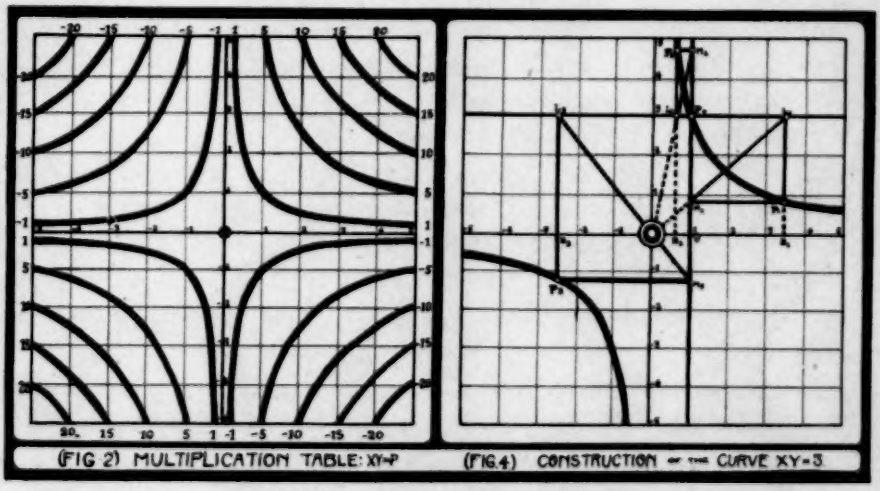
entry S are joined by a curve or straight line, which bears the proper entry once for all in the margin of the table.

3. Then one comes to the notion of a function of position in the plane—the Cartesian co-ordinates X, Y offering merely one mode of



(FIG. 1) ADDITION TABLE: X+Y=S (FIG 3) DIVISION TABLE: $\frac{X}{Y}=Q$

determining position, another important way being by the polar co-ordinates, the radius, and the angle of the circle-ruled paper—or even more generally of a function of position in space or on a curved



(FIG 2) MULTIPLICATION TABLE: XY=P (FIG.4) CONSTRUCTION OF THE CURVE XY=3

surface. These notions, of course, enter at present, and are treated thus graphically especially in work with maps, whether geographical, topographical, or meteorological. We are merely to make mathematical use of these conceptions. Thus, if the graphical addition table (Fig. 1): $X+Y=S$, is taken as the topographic map of a countryside on the horizontal XY plane, S denoting the vertical distance of the point (X, Y, S) above the XY-plane as base-plane, the countryside is a flat hillside, a plane, cutting the horizontal plane $S=0$ in the line $X+Y=0$ and passing through the point $(X, Y, S)=(1, 2, 3)$. Models in clay and other materials corresponding to these topographic maps should be constructed; in classes in physiography this is often done by cutting and pasting. The question of topographic representation of a surface with respect to various (parallel and non-parallel) base-planes should be raised, and especially in the form of the derivation of one representation from another. In this connection should be used the triangle-ruled paper, its three axes diverging at 120° from the chosen origin being taken to correspond to the three mutually rectangular axes of space.

4. Of course, a table of single entry, say for Y as a function of X, is exhibited numerically by two columns of corresponding values of X, Y, and graphically as the single curve made up of the points (X, Y) whose co-ordinates correspond.

By the systematic use of the various kinds of cross-section paper for the construction and interpretation of graphical diagrams of data of whatever origin, it is possible to make our students see and enjoy a rich and suggestive variety of useful functional relations between two or three related variables.

Graphical Computations. Linkages A, A', B

5. In geometry it is customary by elementary processes to construct to scale arithmetic expressions of the form

$$3, \quad \frac{4}{5}, \quad \sqrt{5}, \quad 3 \pm 2\sqrt{5}, \quad \frac{2-3\sqrt{5}}{3-2\sqrt{5}}, \quad \ldots ;$$

that is, of numbers arising from 1 by the sequential application to numbers already at hand of the five processes of addition, subtraction, multiplication, division and extraction of square root of a positive number—and, more generally, of such expressions arising from one

or more numbers $a, b, \ldots$ supposed to be given to scale as linear segments. These constructions involve a large part of the theory of plane geometry and considerable use of the compasses. They are of limited significance, and hence relatively uninteresting.

But *introduce the element of functionality:*—for the variable number X construct and plot to scale the corresponding number Y, where Y denotes one of the various expressions in X,

$$X+3,\ 3X,\ -X+4,\ \frac{1}{X},\ X^2,\ \sqrt{X},\ \sqrt{1-X^2},\ \ldots,$$

and the situation changes immediately in significance and interest.

Use, however, the *constructional methods* of the square-ruled paper, for these methods are, in fact, exceedingly simple and powerful, and of far-reaching significance in theoretical and applied mathematics. Depending on the simplest properties of similar triangles and rectangles (cf. linkages A, A', B below), properties evident after a little experience with the paper, these constructional methods should be introduced in the grades in connection with drawing and observational geometry, and held in focus throughout the high-school and college course.

Indeed, for the purposes of elementary education *our current deductive geometry is of the nature of a fetish*, to be abandoned in favor of a geometry built on a richer system of geometric axioms—a system built to recognize the cross-section paper with its wealth of intuitional relations. In this wider environment there will be abundant need and opportunity for the systematic development of logical power. The matter of the elimination of redundant axioms should be touched on only lightly in secondary-school courses.

6. The cross-section paper is at present used primarily for *plotting*, that is, for recording in graphical form tables of arithmetical data, the result of *arithmetical computations* or of observations or of physical determinations. The constructional methods are in effect methods of *graphical computation*, and this constant checking of arithmetical and algebraic by geometric insight is the source of intense satisfaction, closely akin to that arising in the interaction between scientific theory and experimentation.

Arithmetical computations are so laborious that at present the graphical depiction of functional relations given by formula is of the

nature of a luxury, and I suppose everyone knows how little real significance attaches to the formula by itself. On the other hand, alert students and teachers will by various devices[1] execute with ease the graphical computations[2] of functional expressions.

7. For instruments use merely a straight edge in connection with the two systems of parallel linear scales furnished by the grating of the square-ruled paper, interpolating, if not by eye, by use of paper strip or dividers. But curves once graphically constructed are to be utilized as graphical tables; thus (*a*) the standard hyperbola $XY = 1$ (Fig. 2) is a table of reciprocals Y of numbers X, and of reciprocals X of numbers Y; (*b*) the standard parabola $Y = X^2$ (Fig. 5) is a table of squares Y of numbers X, and of square roots X of numbers Y.

The graphical computation proceeds in scheme step by step parallel to the arithmetical computation. If a computation is impossible arithmetically—e. g., the extraction of the square root of a negative

[1]For instance, in Figs. 4 and 5 are indicated two constructions involving a straight line OLM, which appears in the various examples of each construction as OL_1M_1, OL_2M_2, OL_3M_3, etc. To effect these constructions expeditiously let the straight edge OLM of the ruler rest against a pin at O; then, in Fig. 5, since $UM = OB$, as M runs one by one along the marks of the scale UQ from U, the desired point L is marked on the consecutive vertical lines of the grating from the Y-axis, while, in Fig. 4, as L runs one by one along the marks of the line $Y_L = 3$, with the eye one notes M on UP_0 and transfers by the eye, marking P on the vertical line through L.

Again, curves once constructed can be cut out and used later as rulers for the duplication of such curves perhaps in different positions with respect to the axes. The theory of such transformation of position of curves without change of form, and more generally the theory of transformations with or without change of form, and the corresponding theory of the transformation of the equations of the curves are of central importance.

Again, if a curve is known to be symmetrical with respect to some straight line or point, only one-half of the curve needs to be constructed directly.

Discriminate between the various curves of the same figure by letters for the general points of the respective curves (as below in this paper) and also graphically, by use of dots and dashes, shading, and colors, which should, of course, attach also to the elimination table and linkage diagram, which (as explained below) serve to epitomize the construction.

[2]I take the opportunity to refer to the article "Numerisches Rechnen," by Mehmke, in the *Encyklopädie der mathematischen Wissenschaften* (Leipzig: B. G. Teubner), Vol. I, article F, pp. 941–1081, for information as to the technical aspects of numerical computation, whether arithmetical, or by geometric drawing, or by the use of graphical tables (nomograms), or by apparatus, mechanical or physical. The nomographic methods are rapidly becoming of central importance. A nomogram once constructed for a certain type of problems of numerical computation, any particular

number—so is it graphically; cf. in §16 the construction for $\sqrt{1-x^2}$. If a computation is possible, but the result is not unique, as in the solution of equations, the arithmetical process usually requires tedious approximations, while the graphical process usually exhibits the results immediately in connection with the intersections of curves.

Of course, a computation of either type may be carried through in various ways; a comparison of these ways is interesting and instructive, and in the fundamental cases especially useful as serving to assist in determining a desirable set of geometric axioms.

8. In practice one handles addition directly by means of the linear scales of the paper. (Cf. §16.)

9. We propose the following problem in graphical multiplication. Designating in general the co-ordinates (X, Y) of a point K by X_K, Y_K, (to be read *the X of K, the Y of K*), having given a point P, we seek a point L such that to scale

$$X_L = X_P, \quad Y_L = X_P Y_P.$$

problem of the type is solved by the mere drawing of straight lines and noting the intersections of the lines with one another and with the curves of the nomogram.

Thus, to give a simple example, a nomogram for the solution of the general quadratic equation

$$X^2 + pX + q = 0$$

consists of the ordinary square-ruled paper with the standard parabola $Y = X^2$ (Fig. 5 below). The roots of any particular equation with given p, q are the X's of the points of section with the parabola of the straight line $Y = -pX - q$. This is the line crossing the X- and Y-axes in the respective points $(-\frac{q}{p}, 0)$, $(0, -q)$. One compute $\frac{q}{p}$ arithmetically or graphically (§9 below). Such an auxiliary computation is however unnecessary. Thus, d'Ocagne's nomogram (*loc. cit.*, §46) for the solution of

$$U^2 + pU + q = 0$$

consists merely of three curves (p), (q), (U) bearing three scales for p, q, U respectively, and the straight line joining the scale points p, q cuts the (U) curve in scale-points U, the roots of the equation to be solved. The three scales are, in terms of the square-ruled paper, as follows: the scale for p is the usual scale on the Y-axis; the scale for q is the usual scale on the line $X = 1$; the scale for U is $\left(\frac{1-X}{X}\right)$ on the hyperbola $Y = -\frac{(1-X)^2}{X}$.

Professor C. S. Slichter, of the University of Wisconsin, has arranged various styles of (uniform and non-uniform) ruling of cross-section paper (to be had of dealers in Madison) in such a way that all the elementary problems of logarithmic and trigonometric computation are to be solved merely by the drawing of straight lines.

This problem and naturally the corresponding problem in division: Given a point L, to determine the point P such that

$$X_P = X_L , \quad Y_P = \frac{Y_L}{X_L} ,$$

one solves by similar triangles using the figure $OUBLMP$, which appears in particular cases $OUB, L_1M_1P_1$, $OUB_2L_2M_2P_2$, $OUB_3L_3M_3P_3$ in Fig. 4. We have in this figure in every case

$$X_M = OU = 1 , \quad Y_M = UM = BP = Y_P , \quad X_L = OB = X_P ,$$

and, from similarity of the triangles OUM, OBL,

$$\frac{OU}{OB} = \frac{UM}{BL} = \frac{BP}{BL} ,$$

that is,

$$\frac{1}{X_P} = \frac{1}{X_L} = \frac{Y_P}{Y_L} ,$$

so that indeed

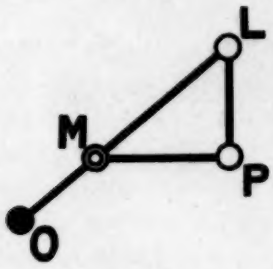
$$Y_P = \frac{Y_L}{X_L} \text{ and } Y_L = X_P Y_P .$$

It is to be noticed in the figure $OUB_3L_3M_3P_3$ how precisely the agreement or rule of signs in the multiplication of two signed numbers fits this geometric situation.

10. Geometric functionality comes to clearer vision by means of the notion of *linkage*—that is, an arrangement of links or bars with slots and pins, fixed or movable in slots, and pivots, and joints, and braces, such that, however the linkage be moved, certain essential properties of the diagram remain. The linkage represents geometrically the general variable diagram and algebraically a system of simultaneous equations between the simultaneous co-ordinates X, Y of the various points of the diagram, from which by elimination the relations sought are to be obtained.

In the *linkage diagrams* of this note points marked in black circles are fixed, those marked in white circles are free to move over the whole plane, those marked in two concentric circles are free to move each along a prescribed line or curve in the plane, while straight lines shown may move but so as to remain straight, and straight lines shown in the X- and Y-directions may move but so as to retain those directions.

Thus the linkage diagram and the *elimination table* for the multiplication figure discussed above are as follows:



$O=(0, 0)$

$X_M=1$

IV. **LINKAGE A**

	L	*M*	*P*
L		$\frac{Y_L}{X_L}=\frac{Y_M}{X_M}$ $\therefore Y_L=X_L Y_M$	$X_L=X_P$ $\therefore Y_L=X_P Y_P$ $\therefore Y_P=\frac{Y_L}{X_L}$
M	——	$X_M=1$	$Y_M=Y_P$
P	——	——	

IV. ELIMINATION TABLE

An elimination table for a diagram involving the variable points *L*, *M*, *P* is an arrangement of six compartments *LL*, *LM*, *LP*, *MM*, *MP*, *PP*, where a compartment—e. g., *MP*—contains the entries connecting the points *M* and *P*, here the entry $Y_M=Y_P$. Here the entries preceded by the sign (∴) for "therefore" are obtained by elimination from the other four entries, which correspond directly to the conditions of the diagram. The entry *MM*, $X_M=1$, specifies the curve (*M*), in this case a line, on which *M* is to move. There are no entries *LL*, *PP*, because the points *L*, *P* are individually unconditioned and free to move over the whole plane

11. Linkage A is a graphical multiplying machine: $Y_L = X_P Y_P$, and a graphical dividing machine: $Y_P = \frac{Y_L}{X_L}$. In any case of multiplication or division, if the two given numbers are not thus given graphically to scale as the co-ordinates (X, Y) of a point P or L, then a preliminary transformation of the data or a suitable modification of the process becomes necessary. Such a transformation would be by means of the lines of the grating and the standard diagonal $Y = X$, (OQ of Fig. 5 below) which throughout plays the rôle of a *turner* of X-segments into equal Y-segments, and *vice versa*.

For instance, if two points H and K are given, with $X_H = X_K$, the point G with

$$X_G = X_H = X_K , \quad Y_G = Y_H Y_K$$

is constructed by means of the figure involving linkage (A) and the turner (T), $Y_T = X_T$, specified by the following relations:

$$Y_H = Y_T = X_T = X_P , \quad Y_K = Y_P , \quad Y_L = Y_G , \quad X_G = X_H = X_K .$$

Construct linkage diagram and elimination table for this figure.

12. Clearly, if in linkage A the point P be made to move along some definite curve, call it the curve (P), the locus of the point P, corresponding to the algebraic relation[1]

$$(P): \ Y_P = \mathbf{P}(X_P) ; \ \text{for instance, } Y_P = X_P ,$$

between the co-ordinates X_P, Y_P, where $\mathbf{P}(X_P)$, (*the P function of the X of P*) denotes in general the algebraic expression in terms of

[1]As a result of the law of the motion of the point P, its co-ordinates X_P, Y_P will be always related in a certain way, expressed algebraically by the fact that Y_P always represents to scale a certain algebraic expression or function $\mathbf{P}(X_P)$ in terms of the co-ordinate X_P as taken to scale. We write for the curve (P),

$$(P): \ Y_P = \mathbf{P}(X_P) ,$$

or as another expression of the same meaning,

$$(P): \ X_P = \mathbf{P}^{-1}(Y_P) .$$

These two equations are read: *the Y of P equals the P function of the X of P; the X of P equals the inverse P function of the Y of P.*

Every curve (P) determines in this way by the functional relations between the co-ordinates X, Y of its variable point P a pair of mutually inverse functions:

$$\mathbf{P}(Z) , \quad \mathbf{P}^{-1}(Z) ,$$

and conversely every pair of inverse functions is representable in this way by a curve drawn to scale with respect to a co-ordinate system. The notations used are intended

X_P which is to scale equal to Y_P for every point P of the curve (P), then the point L will move along some curve (L),

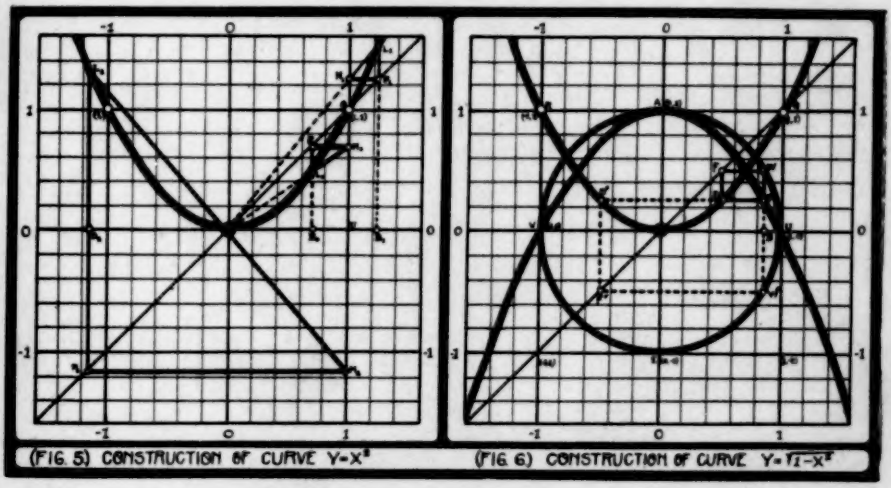
$$(L):\ Y_L=\mathbf{L}(X_L)=X_L\cdot\mathbf{P}(X_L)\ ;\quad \text{in particular, } Y_L=X_L^2\ .$$

Similarly, if L move along (L),

$$(L):\ Y_L=\mathbf{L}(X_L)\ ;\quad \text{for instance, } Y_L=3\ ,$$

then P will move along (P),

$$(P):\ Y_P=\frac{\mathbf{L}(X_P)}{X_P}\ ;\quad \text{in particular, } Y_P=\frac{3}{X_P}\ .$$



(FIG. 5) CONSTRUCTION OF CURVE $Y=X^2$ (FIG. 6) CONSTRUCTION OF CURVE $Y=\sqrt{1-X^2}$

Thus, in particular, we obtain the graphical construction of (Fig. 5) the standard parabola[1] (L): $Y_L=X_L^2$, and (Fig. 4) the rec-

to emphasize the essential equivalence of the curve and the pair of inverse functions.

Given a curve (P) and a point Q; let the vertical line through the point Q meet the curve (P) in the point P_1; and let the horizontal line through the point Q meet the curve (P) in the point P_2. Then $X_Q=X_{P_1}$ and $Y_Q=Y_{P_2}$ so that

$$Y_{P_1}=\mathbf{P}(X_Q)\ ,\quad X_{P_2}=\mathbf{P}^{-1}(Y_Q)\ ,$$

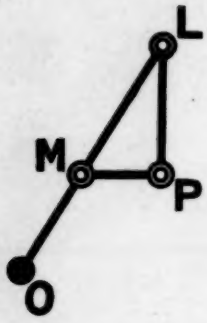
that is, *the Y of P_1 is the P function of the X of Q; the X of P_2 is the inverse P function of the Y of Q.*

Show how by use of the turner to compute graphically from the curve (P) and the point Q the functional expressions

$$\mathbf{P}(Y_Q)\ ,\quad \mathbf{P}^{-1}(X_Q)\ .$$

[1]Students of projective geometry will notice that the metrical constructions of Figs. 4 and 5 are precisely the Pascal hexagon*projective constructions for conics,

tangular hyperbola (P): $X_P Y_P = 3$, with the following elimination tables and linkage diagrams:



V. **LINKAGE A'** (FIGS 5 AND 4)

$O=(0,0)$	$O=(0,0)$
$X_M=1$	$X_M=1$
$Y_P=X_P$	$Y_L=3$
$\therefore Y_L=X_L^2$	$\therefore X_P Y_P=3$
$(V,$ Fig. 5$)$	$(V,$ Fig. 4$)$

	L	M	P
L	$\therefore Y_L=X_L^2$	$Y_L X_M=X_L Y_M$	$X_L=X_P$
M	——	$X_M=1$	$Y_M=Y_P$
P	——	——	$Y_P=X_P$

(V, FIG. 5) PARABOLA $Y_L=X_L^2$

	L	M	P
L	$Y_L=3$	$Y_L=X_L Y_M$	$X_L=X_P$
M	——	$X_M=1$	$Y_M=Y_P$
P	——	——	$\therefore X_P Y_P=3$

(V, FIG. 4) HYPERBOLA $X_P Y_P=3$

being given three points and tangent lines at two of the three points, viz. (Fig. 4), the point P_0 and the X- and Y-axes as asymptotes; (Fig. 5) the point Q and the X-axis and the line at infinity as tangents at their respective intersections with the Y-axis.

13. $\mathbf{L}(Z)=Z\cdot\mathbf{P}(Z)$.—This linkage A' yields, as stated above, more general curves for P, L, respectively, if L, P are made to traverse more general curves, viz.:

V. **LINKAGE A' (GENERAL) $\mathbf{L}(Z)=Z\cdot\mathbf{P}(Z)$**

$$O=(o,\ o) \qquad O=(o,\ o)$$
$$X_M=1 \qquad X_M=1$$
$$Y_P=\mathbf{P}(X_P) \qquad Y_L=\mathbf{L}(X_L)$$
$$\therefore Y_L=X_L\cdot\mathbf{P}(X_L)=\mathbf{L}(X_L) \qquad \therefore Y_P=\frac{\mathbf{L}(X_P)}{X_P}=\mathbf{P}(X_P)$$

For instance, if P traverses the quadratic parabola $Y=X^2$ of Fig. 5, L traverses the cubic parabola $Y=X^3$; if P traverses this parabola, L traverses the quartic parabola $Y=X^4$. Thus, by applying linkage A', being given the curve (P) and finding the curve (L), in succession starting from the standard diagonal $Y=X$ we obtain the curves $Y=X^2$, $Y=X^3$, $Y=X^4$, ... and, in general, $Y=X^n$ for n a positive integer.[1] The first quadrant of this curve serves for the whole curve, since the curve is symmetrical with respect to the Y-axis (n even) or the origin (n odd), and, it serves also, by interchanging the rôles of the X- and Y-axes, for the curve $X=Y^n$, that is, for the curve $Y=X^{\frac{1}{n}}$.

14. $\mathbf{G}(Z)=\mathbf{H}(Z)\cdot\mathbf{K}(Z)$.—Similarly, by means of the linkage of §11, we construct from two given curves (H), (K):

$$Y_H=\mathbf{H}(X_H)\ ,\quad Y_K=\mathbf{K}(X_K)\ ,$$

the curve (G):

$$Y_G=\mathbf{H}(X_G)\cdot\mathbf{K}(X_G)\ ,$$

viz., *the product as to Y of the two curves (H) (K), in the sense*

$$X_G=X_H=X_K\ ,\quad Y_G=Y_HY_K\ ,$$

so that

$$\mathbf{G}(Z)=\mathbf{H}(Z)\cdot\mathbf{K}(Z)\ ,$$

where Z denotes an arbitrary number the basis of the functional expressions **G, H, K.**

Of course, the same linkage otherwise employed yields from the curves (G) and (H) the curve (K) as the quotient as to Y of (G) by (H).

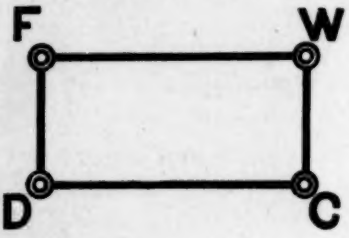
[1]The general case, n fractional, positive or negative, is treated in §21.

15. The linkage of §11 specializes to linkage A if $H=T$, and accordingly $K=P$, $G=L$. We have (cf. §13)

$$X_T=X_P=X_L\,,\quad Y_L=X_PY_P=Y_TY_P\,,$$

so that the curve (L) is the product as to Y of the curve (P) and the turner (T).

16. The rectangular linkage B (VI) is of central importance in the compounding of curves and functions. Three points C, D, F traverse given curves (C), (D), (F), and then W traverses a certain



$$Y_W=Y_F\,,\quad X_F=X_D\,,$$
$$Y_D=Y_C\,,\quad X_C=X_W\,.$$

VI. **LINKAGE B**

curve (W).

Thus, if D traverses the turner $Y_D=X_D$ and F the straight line $Y_F=3\cdot X_F$ and C a curve $Y_C=\mathbf{C}(X_C)$, then

$$Y_W=Y_F=3\cdot X_F=3\cdot X_D=3\cdot Y_D=3\cdot Y_C=3\cdot\mathbf{C}(X_C)=3\cdot\mathbf{C}(X_W),$$

so that this use of linkage B effects the transformation of the curve (C) into a curve (W) as follows:

$$X_W=X_C\,,\quad Y_W=3\cdot Y_W\,,$$

and correspondingly constructs from a function $\mathbf{C}(Z)$ a function $\mathbf{W}(Z)$ where

$$\mathbf{W}(Z)=3\cdot\mathbf{C}(Z)\quad.$$

Similarly, arrange the linkage to construct from a function $\mathbf{P}(Z)$, graphically given by a curve $(\mathbf{P})$: $Y_P=\mathbf{P}(X_P)$, the various functions,

$$\mathbf{W}(Z)=2\cdot\mathbf{P}(Z)\,,\ \tfrac{1}{2}\mathbf{P}(Z)\,,\ \mathbf{P}(2Z)\,,\ \mathbf{P}(\tfrac{1}{2}Z)\,,$$

and

$$\mathbf{W}(Z)=\mathbf{P}(Z)+2\,,\ \mathbf{P}(Z)-2\,,\ \mathbf{P}(Z+2)\,,\ \mathbf{P}(Z-2)\quad.$$

In these cases D traverses the turner and either C or F the curve (P), while the third point F or C traverses a straight line.

17. To apply linkage B to construct (Fig. 6) the circle

$$Y_W=\sqrt{1-X_W^2}$$

of radius 1 and center at origin, let F traverse the turner $Y=X$, and D the standard parabola $Y=X^2$, $X=\sqrt{Y}$, and C the parabola[1] $Y=1-X^2$, that is,

$$Y_F=X_F\,,\quad X_D=\sqrt{Y_D}\,,\quad Y_C=1-X_C^2\,,$$

so that indeed

$$Y_W=Y_F=X_F=X_D=\sqrt{Y_D}=\sqrt{Y_C}=\sqrt{1-X_C^2}=\sqrt{1-X_W^2}\,.$$

Construct this circle

$$Y_W=\sqrt{1-X_W^2}\,,\quad X_W^2+Y_W^2=1$$

by the same linkage also as follows:

$$X_D+Y_D=1\,,\quad Y_C=X_C^2\,,\quad X_F=Y_F^2\,;$$

so that indeed

$$1=X_D+Y_D=X_F+Y_C=Y_F^2+X_C^2=Y_W^2+X_W^2\,.$$

The curve (D) is in this case the straight line AU of Fig. 6.

18. $\mathbf{W}(Z)=\mathbf{F}\mathbf{D}^{-1}\mathbf{C}(Z)$.[2]—The curve (W) is in a certain sense compounded out of the curves (C), (D), (F). Just so the two mutually inverse functions $\mathbf{W}(Z)$, $\mathbf{W}^{-1}(Z)$ associated with the curve (W) are compounded out of the functions associated with the curves (C), (D), (F). For, in the customary notations, with

$$Y_W=\mathbf{W}(X_W),$$

we have

$$\begin{aligned}Y_W=Y_F&=\mathbf{F}(X_F)=\mathbf{F}(X_D)=\mathbf{F}\big(\mathbf{D}^{-1}(Y_D)\big)\\&=\mathbf{F}\big(\mathbf{D}^{-1}(Y_C)\big)=\mathbf{F}\Big(\mathbf{D}^{-1}\big(\mathbf{C}(X_C)\big)\Big)\\&=\mathbf{F}\Big(\mathbf{D}^{-1}\big(\mathbf{C}(X_W)\big)\Big)\,,\end{aligned}$$

that is, in briefer form,

$$Y_W=\mathbf{W}(X_W)=\mathbf{F}\mathbf{D}^{-1}\mathbf{C}(X_W)\,,$$

so that as operator on a general number Z the function $\mathbf{W}$ is the

[1] Obtained from $Y=X^2$ by reflection on X-axis and by lifting 1 in the direction of the Y-axis. Make this curve $Y=1-X^2$ also by use of linkage B as follows:

$$Y_F=1-X_F\,,\quad X_D=Y_D\,,\quad Y_C=X_C^2\,,$$

so that

$$Y_W=Y_F=1-X_F=1-X_D=1-Y_D=1-Y_C=1-X_C^2=1-X_W^2\,.$$

[2] To be read, the W function of Z equals the F function of the inverse D function of the C function of Z. This is the composition of functions effected in general by linkage B.

function resulting from the composition of the functions $\mathbf{C}$, $\mathbf{D}^{-1}$, $\mathbf{F}$ in that order, viz.

(*a*) $$\mathbf{W}(Z)=\mathbf{F}\mathbf{D}^{-1}\mathbf{C}(Z)\,.$$

Similarly,

(*b*) $$\mathbf{W}^{-1}(Z)=\mathbf{C}^{-1}\mathbf{D}\mathbf{F}^{-1}(Z)\,.$$

The formula (*a*) was just now derived from the linkage relations between the curves (C), (D), (F), (W) by process of algebraic elimination to express Y_W in terms of X_W.

It is desirable to see conversely how the linkage enables us to compute graphically by means of (C), (D), (F) for every number Z a corresponding number $\mathbf{W}(Z)=\mathbf{F}\,\mathbf{D}^{-1}\,\mathbf{C}(Z)$. Taking the Z as an X, say X_C, we have $\mathbf{C}(Z)$ given as

$$\mathbf{C}(Z)=\mathbf{C}(X_C)=Y_C\,,$$

and then

$$\mathbf{D}^{-1}\mathbf{C}(Z)=\mathbf{D}^{-1}(Y_C)=\mathbf{D}^{-1}(Y_D)=X_D\,,$$

and then

$$\mathbf{W}(Z)=\mathbf{F}\mathbf{D}^{-1}\mathbf{C}(Z)=\mathbf{F}(X_D)=\mathbf{F}(X_F)=Y_F\,.$$

Thus, to express Y_W in terms of X_W in the form $Y_W=\mathbf{F}\mathbf{D}^{-1}\mathbf{C}(X_W)$, we proceed around the rectangle in the sense $W\,F\,D\,C\,W$, and conversely, to compute graphically from given X_W the expression $\mathbf{F}\mathbf{D}^{-1}\mathbf{C}(X_W)$ as Y_W we proceed around the rectangle in the opposite sense $W\,C\,D\,F\,W$.

19. $\mathbf{W}(Z)=\mathbf{D}^{-1}(Z)$.—If C and F traverse the turner $Y=X$, then D and W are always symmetrical with respect to the turner, and

$$X_W=X_C=Y_C=Y_D\,,\quad Y_W=Y_F=X_F=X_D\,,$$

so that linkage B transforms the curve (D),

$$(D):\ Y_D=\mathbf{D}(X_D)\,,$$

into the curve (W) symmetrical to (D) with respect to the turner, viz.:

$$(W):\ X_W=\mathbf{D}(Y_W)\,,\ \text{i. e.,}\ Y_W=\mathbf{D}^{-1}(X_W)\,.$$

Here

$$\mathbf{W}(Z)=\mathbf{D}^{-1}(Z)\,,$$

so that this is a construction for the function $\mathbf{W}(Z)$ inverse to a given function $\mathbf{D}(Z)$.

The present relation $\mathbf{W}(Z)=\mathbf{D}^{-1}(Z)$ is in accord with the general

relation $\mathbf{W}(Z)=\mathbf{F}\,\mathbf{D}^{-1}\,\mathbf{C}(Z)$ of §17, since in the present case for every number Z $\mathbf{F}(Z)=Z$ and $\mathbf{C}(Z)=Z$.

20. $\mathbf{W}(Z)=\mathbf{FC}(Z)$ or $\mathbf{FD}^{-1}(Z)$ or $\mathbf{D}^{-1}\mathbf{C}(Z)$.—If D traverses the turner, so that $\mathbf{D}^{-1}(Z)=Z$, then linkage B constructs from given functions $\mathbf{C}(Z)$, $\mathbf{F}(Z)$ the compound function

$$\mathbf{W}(Z)=\mathbf{FC}(Z)\,.$$

Similarly, if C or F traverses the turner, we construct the compound function

$$\mathbf{W}(Z)=\mathbf{FD}^{-1}(Z) \text{ or } \mathbf{D}^{-1}\mathbf{C}(Z)\,.$$

21. The curve $Y=X^k$, associated with the function Z^k where $k=\frac{n}{d}$, n and d positive relatively prime integers, is constructed by linkage B from the curves:

$$(C)\quad Y_C=X_C^n;\qquad \mathbf{C}(Z)=Z^n\,,$$

$$(D)\quad Y_D=X_D^d;\quad \mathbf{D}^{-1}(Z)=\frac{1}{Z^d}\,,$$

which were constructed in §13, while F traverses the turner, for by §20

$$\mathbf{W}(Z)=\mathbf{D}^{-1}\mathbf{C}(Z)=(Z^n)^{\frac{1}{d}}=Z^{\frac{n}{d}}=Z^k\,,\quad Y_W=\mathbf{W}(X_W)=X_W^k\,.$$

The curve $Y=X^{-k}=X^{-\frac{n}{d}}$ may be constructed from the same curves (C) (D) by letting F traverse the standard rectangular hyperbola

$$(F):\ Y_F=\frac{1}{X_F}\,;\quad \mathbf{F}(Z)=\frac{1}{Z}\,,$$

(cf. §12, Fig. 4), for

$$\mathbf{W}(Z)=\mathbf{FD}^{-1}\mathbf{C}(Z)=\frac{1}{Z^{\frac{n}{d}}}=Z^{-\frac{n}{d}}=Z^{-k}\,,$$

$$Y_W=\mathbf{W}(X_W)=X_W^{-k}\,.$$

Construct it also thus:

$$\mathbf{F}(Z)=\frac{1}{Z}\,,\quad \mathbf{D}(Z)=Z\,,\quad \mathbf{C}(Z)=Z^k\,,$$

so that indeed

$$\mathbf{W}(Z)=\mathbf{FC}(Z)=\frac{1}{Z^k}=Z^{-k}\,.$$

22. The system of curves $Y=X^k$, $k=\pm\frac{n}{d}$, is worthy of remark. All the curves contain the point (1, 1) and run through the first quadrant $X>0$, $Y>0$ of the plane. We consider their distribution in the first quadrant. Those with k positive contain the origin (0, 0), those with k negative are asymptotic to both the co-ordinate axes. These two classes $(k=+)$, $(k=-)$ are separated by the curves $(k=0)$: $Y=1$, $(k=\infty)$: $X=1$, which partition the quadrant into four (call them) quarters; the class $(k=+)$ lies in the SW and NE quarters (to speak geographically as from the point (1, 1)), while the class $(k=-)$ lies in the SE and NW quarters of the quadrant. The curves $(k=\pm)$ fall into two classes $(k=\pm>)$, $(k=\pm<)$ according as k is numerically greater than or less than 1. The turner $(k=+1)$: $Y=X$ subdivides the class $(k=+)$, while the standard hyperbola $(k=-1)$: $Y=X^{-1}$ subdivides the class $(k=-)$. There are thus eight octants of the first quadrant. The curves $(k=+>)$ occupy the SSW and NNE octants, and come into the origin tangentially to the X-axis; the curves $(k=+<)$ occupy the WSW and ENE octants, and come into the origin tangentially to the Y-axis. The area inclosed by the turner, either co-ordinate axis, and the hyperbola $(k=-1)$ is in fact infinite. The curves $(k=-<)$ occupy the WNW and ESE octants, and inclose with the Y-axis a finite, and with the X-axis an infinite, area. The curves $(k=->)$ occupy the NNW and SSE octants, and inclose with the Y-axis an infinite, and with the X-axis a finite, area. These statements concerning areas are capable of elementary proof. The curves $Y_A=X_A^k, Y_B=X_B^{-k}$ are (cf. §14) as to Y reciprocals each of the other, since $X_A=X_B$ implies $Y_AY_B=1$; and they are likewise as to X reciprocals each of the other, since $Y_A=Y_B$ implies $X_AX_B=1$.

The march of the curves $Y=X^k$ in the other three quadrants of the plane depends upon the evenness—oddness character of the numerator and denominator of $k=\pm\frac{n}{d}$. The curves $\left(k=\frac{\text{even}}{\text{odd}}, \frac{\text{odd}}{\text{odd}}, \frac{\text{odd}}{\text{even}}\right)$ are symmetrical respectively as to the Y-axis, the origin, the X-axis, and pass from the first quadrant into the second, the third, the fourth quadrant.

23. By use of the curves $Y=X^k, Y=X^l$, each of the constructions (§17) of the circle by linkage B may be generalized, giving the curve

$$Y=\sqrt[l]{1-X^k}\,,\quad X^k+Y^l=1\,,$$

viz., in the first place,

$$Y_W=Y_F=X_F=X_D=\sqrt[l]{Y_D}=\sqrt[l]{Y_C}=\sqrt[l]{1-X_C^k}=\sqrt[l]{1-X_W^k}\,,$$

and, in the second place,

$$1=X_D+Y_D=X_F+Y_C=Y_F^l+X_C^k=Y_W^l+X_W^k\,.$$

The second construction is the more symmetrical. In it the curve $X_W^k+Y_W^l=1$ arises by transformation via the curves

$$Y_C=X_C^k\,,\quad X_F=Y_F^l$$

out of the straight line (AU of Fig. 6)

$$Y_D+X_D=1\,.$$

24. More generally, any curve

$$\mathbf{C}(X_W)+\mathbf{E}(Y_W)=1$$

arises by transformation via the curves

$$Y_C=\mathbf{C}(X_C)\,,\quad X_F=\mathbf{E}(Y_F)=\mathbf{F}^{-1}(Y_F)$$

out of the straight line

$$Y_D+X_D=1\,;$$

and similarly, in general, any curve

$$\Delta\big(\mathbf{C}(X_W)\,,\ \mathbf{E}(Y_W)\big)=0$$

is constructed from the curve

$$\Delta(Y_D\,,\ X_D)=0$$

by means of the curves

$$Y_C=\mathbf{C}(X_C)\,,\quad X_F=\mathbf{E}(Y_F)\,,$$

viz.;

$$0=\Delta(Y_D\,,\ X_D)=\Delta(Y_C\,,\ X_F)=\Delta\big(\mathbf{C}(X_C)\,,\ \mathbf{E}(Y_F)\big)=\Delta\big(\mathbf{C}(X_W)\,,\ \mathbf{E}(Y_W)\big)\,.$$

25. The reader perceives, or after a little practice will perceive, how readily the linkages A' and B alone or suitably combined serve to replace arithmetical by graphical computation of functional expressions,—and it is hoped that he will join with the writer in the following

Concluding Agreements

Today there is general agreement to cut out of arithmetic and algebra many complicated features of limited significance. We make the same agreement as to geometry.

In general, we agree to develop arithmetical, algebraic, geometric technique in a physical and intellectual environment logically and psychologically rich, full of movement, force, color, full of connotations and implications of and for real life of all kinds, including most certainly the real life of mathematics and the sciences.

To secure for our young students a tolerable appreciation of the civilization of the twentieth century on the side of the theoretical and applied mathematical sciences, as teachers of mathematics we agree to shape our instruction in mathematics from the beginning from a point of view no older and no lower than that of the wonderful seventeenth century, and to this end, speaking theologically, we propose to

Canonize the Cross-Section Paper.

AN EXPERIMENT ON THE MAGNIFYING POWER OF A SIMPLE LENS.

By R. A. Millikan,
University of Chicago.

The putting together of simple convex lenses so as to form telescopes and microscopes is a type of exercise of which many of our laboratory courses are much in need. But if a student is to gain any real insight into the way in which a telescope or microscope performs its functions he ought to make a rough determination of magnifying power and compare it with the value computed from the focal lengths of the two lenses which he uses. The following experiment is introduced into the University of Chicago list in order that the student may appreciate the fact that the lens which he uses as an eyepiece has a perfectly definite magnifying power, i. e., that it magnifies the image which is formed at its focus a definite number of times. The experiment lens is used to best advantage as a magnifier, it is brought as near as possible to the eye and the object is placed in its focal plane, and also that when so used the size of the image formed upon the retina is as many times larger than the image formed when the object is viewed with the unaided eye, at the distance of most distinct vision, namely, 25 cm., as 25 cm. is times larger than the focal length of the lens, in other words, that the magnifying power of a simple lens is $25 \div F$.

Each student is furnished with a so-called "linen tester,"* a simple lens about 1 cm. in diameter, at the focus of which is a square hole in the brass frame which serves as the support for the lens. With this he performs an experiment of which the following are the directions:

"Lay one meter stick on the table, and with the aid of another one held vertically, adjust the position of the eye which is viewing the horizontal stick so that the distance from the stick to the eye is just 25 cm. Then, keeping the head always in this position, bring up the lens as close as possible to the other eye. Keep both eyes open at the same time and observe how many millimeters on the stick seen with one eye are covered by the hole seen through the lens with the other eye. Divide the number by the measured width of the hole in mm. This is obviously the magnifying power of the simple lens, since it is how many times larger the object appears when seen through the lens than when viewed with the naked eye at the distance of most distinct vision, viz., 25 cm. Measure as accurately as possible the focal length F of the lens, i. e., the distance from the middle of the lens to the hole and see how well the observed magnifying power agrees with the theoretical value, $25 \div F$.

The agreement between the observed and calculated values in this experiment is invariably good. For example, the observed magnifying power will be 12.5 while the calculated may be 13. The difference is seldom larger than this.

We use two of these linen testers whose magnifying powers have been found in this way, for the objective and eyepiece of a compound microscope, simply attaching them by means of rubber bands to the two ends of a tube about 5 inches long. The magnifying power of such an arrangement is about 40. We also use one of the linen testers for the eyepiece of the telescope which we have the students construct. Altogether these linen testers are the most convenient, most inexpensive, and most serviceable part of our optical laboratory equipment.

* These can be had at the department stores at about 25 cents each.

SOME PROPERTIES OF MATTER AT LOW TEMPERATURES.*

By Prof. J. S. Shearer,

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One of the most important elements which influence man's physical existence is the temperature of his surroundings, the very maintenance of life being conditioned on a comparatively narrow temperature range. The change from winter's cold to summer heat undoubtedly gave to prehistoric man his ideas regarding limits of temperature variation, while acquaintance with fire, and in certain parts of the earth with volcanic eruptions and geysers perhaps gave an extension as regards the question of higher temperature. We are still, however, accustomed to consider any temperature as low which falls very much below the temperature at which ice melts, even though our experience is now extended very considerably beyond that limit. The processes of manufacture which often demand the use of very high temperatures have enlarged our knowledge in the direction until they have become somewhat familiar to most observers. Yet, beyond the reduction secured by ordinary freezing mixtures, temperatures considered as low are by no means familiar to most of us. We may therefore assume that any temperature more than 30 deg. on the centigrade scale below the freezing point of water may come properly under the definition of low.

The methods employed in the production of these temperatures may be divided into three distinct classes. By the addition of suitable salts to solids it is possible to hasten the process of liquefaction; the heat required to change from solid to liquid state being abstracted from the surroundings gives a temperature reduction. If we use, for example, various salts with ice in proper proportions the temperature of the melting mixture may be reduced to perhaps twenty or thirty degrees below the zero point, depending upon the nature of the mixture. It is not, in general, convenient to use this process with many solids for various reasons. If, however, we choose a mixture of solid carbon-dioxide and ether in proper proportions a temperature of approximately—80 deg. C may be secured. Another natural process requiring the

* This is an abstract of a lecture given at the December, 1905, meeting of the New York State Science Teachers' Association. It was demonstrated with the aid of liquid air.

expenditure of heat consists in the vaporization of a liquid. The conditions under which vaporization continues are more or less under control. Since part of the work done in changing a liquid to a gas consists in overcoming the external pressure which opposes the increased volume from the liquid to the gaseous state, a reduction of external pressure will facilitate the process of vaporization. Increased vaporization, however, demands expenditure of heat and if the process be maintained the temperature of both the liquid and its surroundings will be reduced by the continued change of state. If it were possible to secure a series of liquids whose boiling points were properly spaced a series of controllable temperatures might be secured by boiling these liquids at varying pressures. By a combination of cooling with solid carbon-dioxide and the application of pressure, early investigators succeeded in liquefying certain gases which had previously been regarded as existing only in the vapor state, and by boiling these liquids under reduced pressure a still greater reduction in temperature was secured.

It at first appeared to investigators in this field that since the conditions which determine the physical state of substance are the pressure and temperature to which the body is subjected and since increase of pressure as well as reduction in temperature always tends to change vapors into liquids, that it would be possible by a sufficient increase in pressure to change the most refractory gases into the liquid form. The researches of Andrews showed that such was not the case. He demonstrated that unless a gas be reduced in temperature below a point which is definite and fixed for each particular substance it would be impossible by any increase of pressure to change the material to the liquid state. For example, it would be absolutely impossible to liquefy carbon-dioxide unless the temperature of the gas was reduced below about 34 deg. C. This point, to which the temperature must be reduced before pressure can cause liquefaction, is spoken of as the critical point for that particular substance. It should be noted, however, that simple reduction of temperature would not necessarily produce liquefaction, since to each temperature below the critical one, there corresponds a definite vapor pressure. The further the gas is reduced, however, below its critical temperature the smaller will be the applied pressure necessary to produce liquefaction. The critical temperature

of oxygen, nitrogen and hydrogen is much below the lowest temperature attained by boiling ethelene and similar liquids under reduced pressure, so that for many years these came to be named permanent gases and greater temperature reduction was required before they could be liquefied. Another process which results in lowering of temperature is the expansion of a gas against an external pressure. The process of compression of any gas results in the production of a rise in temperature, while the same gas when expanded usually cools. By a combination of the processes above described, Olzewski, Dewar and others succeeded in liquefying oxygen, nitrogen and air. The first plants arranged for this purpose were, however, exceedingly complicated and expensive. A few years ago Professor Linde of Germany and Dr. Hampson, an English physicist, devised apparatus for utilizing continuously the cooling of gases by expansion from high pressures so as to produce temperatures much below the critical temperature for nitrogen or oxygen, in fact lowering the temperature to such a point that these gases would liquefy at ordinary atmospheric pressure. By exhausting the gas rapidly from the vessel containing these boiling liquids a still further reduction of temperature was available.

Let us now consider for a moment the question of probable limits of available temperatures. As regards the higher limit we are not likely to secure a temperature much above that of the positive carbon in an electric arc, especially when designed for electric furnace operation. This temperature has, in reality, never been measured by ordinary methods but is probably somewhere between three and four thousand degrees on the centigrade scale. Our idea of the lowest temperature which we might expect to attain is based on the supposition that the pressure exerted by a gas upon the enclosing walls reduces by 1-273 of its value at 0 deg. for each degree of temperature reduction. If then the gas should *remain* a *gas* and should *maintain* the same rate of pressure reduction as the temperature was still further decreased we should expect that at a temperature of -273 the gas would be incapable of exerting any expansive pressure whatever. This supposition is not one which is warranted by our experience with any of the common gases since they all change from the gaseous to the liquid state at ordinary pressures before a temperature approximately of 273 below zero is reached with the exception of the

rare gas helium. A picture of the range of temperatures below zero degrees may be shown by measuring a string 273 units in length as attached here to the ceiling and considering the point of attachment as zero on the centigrade scale. The first tag attached corresponds to carbon-dioxide mixed with ether, or 80 deg. C. The next corresponds to the critical temperature for atmospheric air very approximately 130 deg. below zero. The next is the boiling point of oxygen 182.4 and slightly below that the boiling point of nitrogen -195.6.

The next step in the development of low temperatures was one which required careful consideration and a further application of the methods which have already been mentioned. The critical temperature of hydrogen is lower than can be attained by the boiling of nitrogen under reduced pressure, that is, much lower than -210 deg. C. (This temperature has recently been determined and is very closely -240.8.) The Hampson process of liquefaction has been successfully applied not only to the case of oxygen and nitrogen but also to the case of hydrogen, and because of its simplicity and effectiveness it is coming to be regarded as the standard method for the liquefaction of gases having low boiling points. The general principle of the apparatus may be illustrated as follows:

Suppose we consider a long metal tube having an adjustable valve at one end and connected at the other with a powerful air compressor capable of supplying a considerable quantity of air at a pressure of 3,000 pounds per square inch. If the valve at the end of our pipe be opened expansion to atmosphere pressure will entail a reduction of temperature. This temperature reduction, however, will not be extremely great and would be largely neutralized by the inflow of heat from the surroundings. Suppose, however, that the tube is insulated as far as possible from all heat supply and is enclosed in a second tube so that the gas partially cooled by expansion passes back on the outside of the first tube, regaining its temperature by absorbing heat from the tube conveying the incoming gas. In this way the gas at the expanding nozzle will be gradually reduced in temperature until finally it comes below the temperature at which liquefaction will take place at atmospheric pressure. As a matter of convenience in installation and operation, the tube carrying the gas is wound up in a spiral coil and is carefully protected from external heat-

ing by a suitably constructed jacket. Such a liquefier will operate successfully in the case of air without preliminary cooling but in the liquefaction of hydrogen the gas under pressure must be cooled somewhat below 80 deg. C before liquefaction can be produced. The effective cooling is due to the slight mutual attraction of gas particles, i. e. to the Joule-Thompson Effect.

The measurement of low temperatures can be accomplished by the use of gas thermometers, thermo-elements, resistance thermometers, or by the use of certain special liquids whose freezing points are extremely low. For the lowest temperatures now attainable, however, a helium gas thermometer or a properly constructed thermo-element is the most practical.

Undoubtedly one of the most interesting substances at low temperatures is air in liquid form. We have in this small beaker a non-viscous, pale blue, liquid whose density is approximately unity, as is readily shown by pouring some on the surface of water and observing the immersion. The surface tension of the liquid is small and the temperature of this particular specimen is probably very close to —188 deg. on the centigrade scale. The temperature varies according to the relative proportion of oxygen and nitrogen in the mixture. The density also varies with the composition from 1.31 for oxygen to .791 for pure nitrogen. Liquid oxygen is slightly magnetic, as may be shown by observing the attraction of a small horse-shoe magnet for the liquid. Liquid air, oxygen or nitrogen behave precisely as any other liquids, excepting that they happen to have boiling temperatures at atmospheric pressure very much below those to which we are ordinarily accustomed. The behavior of liquid air when poured upon a surface at room temperature is exactly the same as in the case of water placed upon a surface extremely hot as compared with the boiling point of water. A layer of gas is developed between the hot plate and the cold liquid which forms a cushion preventing actual contact between the two. If we pour liquid air upon a small glass in the vertical projection field of the lantern we will note that it vibrates quietly for some time without any sputtering or hissing until it finally comes in contact with the glass when it boils much more violently. In common with all other liquids, liquid air has a definite heat of vaporization and it requires the expenditure of approximately 51 calories of heat to convert one gram of liquid air at its boiling point to gaseous air at

the same temperature. If we consider the specific heat of air as contant between its boiling point and 0 deg. C, the total heat required to convert one gram of liquid air at its boiling point to ice cold air would be nearly 95 calories.

In order to keep the liquid it is necessary to construct vessels which will prevent acquisition of heat, since if the heat of vaporization is not supplied, the air will retain the liquid form. The most successful vessels for this purpose were invented by Professor Dewar of the Royal Institution, London, and are known as Dewar bulbs. They are constructed of glass instead of metal, which prevents the conduction of heat from the surroundings. They are made double walled and the air between the two walls is removed so as to prevent the transfer of heat from the outside to the liquid by conduction or convection of the gas and finally they are frequently coated with a layer of silver which serves to reflect radiant heat and light, thus very effectively preventing heat transfer to the liquid.

Since no pressure, however great, would keep air liquid at a temperature above -130 deg. vessels containing the liquid must not temperature above --130 deg. vessels containing liquid air must not be tightly closed. Pouring a small quantity into a piece of heavy rubber tubing and closing both ends the pressure developed on evaporation is rapidly shown.

By the use of liquid air as a cooling agent, it is possible to liquefy many gases at comparatively low pressures. Taking a tube containing ethelene which in the field of the lantern is apparently empty, and immersing it for a moment in a bath of liquid air, we find that we have an accumulation of liquid 1 or 2 cm. deep in the bottom. Were the earth to be cooled to a temperature of --195 deg. C we should have a layer of liquid upon its surface approximately 35 ft. in depth with various solids not especially familiar to us, such as solid carbon dioxide, ethlene, etc., floating therein.

If we consider the effect of great reduction of temperature upon liquids, we find that at the temperature of liquid air, mercury, alcohol, ether, chloroform, etc., have become solids. A test tube filled with mercury gives us a solid which will persist for a considerable time after removal from the liquid and which you see is malleable and not unlike soft lead in appearance. Certain liquids such as pentane and petroleum-ether have freezing points below the temperature of liquid air and may therefore be

used in thermometers for low temperature measurement. If we take ordinary coal gas and pass it through a tube immersed in liquid air and observe the character of the flame at the opposite end of the tube we will note that its luminous qualities are greatly reduced, ultimately giving a flame which chemists would recognize as characteristic of burning hydrogen. Cutting off the gas supply and removing the tube from the cooling liquid, we observe that a considerable amount of solid material is present in the bottom of the tube and on the application of heat this solid material changes to a combustible vapor which may be lighted at both ends of the tube and giving a highly luminous flame. Liquid air contains a higher percentage of oxygen than ordinary gaseous air, owing to the fact that the boiling point of oxygen is higher than that of nitrogen and consequently a greater per cent of the oxygen liquefies than of the nitrogen. This fact may be shown by observing the ability to support combustion of vapor from an old sample of liquid air, which has become still richer in oxygen by the more rapid vaporization of the nitrogen, in which ordinary wool burns with explosive violence. Saturated charcoal behaves on ignition very much as gunpowder.

When we come to consider the effect of reduced temperature upon solids, we may say that brittleness, rigidity and tensile strength are in general increased. If we take a small coil of fuse wire incapable of supporting a given weight at ordinary temperatures and retaining its spiral form, we find that upon reducing to the temperature of liquid air the weight is readily sustained, and we may note the gradual straightening out of the coil as the temperature rises. Flowers, beefsteak, and pieces of rubber become extremely brittle at these temperatures, a piece of rubber tubing having elastic properties similar to a piece of glass.

A further peculiar effect may be noted with regard to ability of bodies to store up luminous energy and give it out as light after the exciting source has been removed. Take for example, an egg which has been subjected to liquid air temperature, also a piece of paraffin candle and a tube of solid alcohol and expose them for a moment to the radiation from an arc lamp and then observe them in a dark room. Each glows with a characteristic phosphorescent light, the egg especially giving a beautiful wavering blue color. It might be of interest to observe in this connection that liquid air like ordinary air is a good insulator and that we may have in the same small beaker simultaneously one

of the highest and one of the lowest temperatures conveniently attainable, by operating an electric arc beneath the surface of liquid air. Notice that the arc is extremely brilliant and is maintained without serious difficulty.

I wish to call your attention to the fact that the temperatures here exhibited are by no means the lowest which have yet been attained. The following table may be useful as giving certain important temperature points for reference.

Solid CO_2 + Ether	-79.8 deg. C.
Boiling Oxygen at a pressure of one atmosphere.	-182.4
Boiling Nitrogen at a pressure of one atmosphere	-195.5
Boiling O at a pressure of one mm. of mercury.	-228.
Critical point of air, (approx.)	-130.
Critical point of Hydrogen	-240.8
Boiling point H	-252.5
Melting point of H	-257.
Lowest temperature attainable by use of H, (estimated)	-260.
Lowest temperature as yet actually measured..	-258.5
Estimated temperature by expanding helium cooled to -258	-266.5

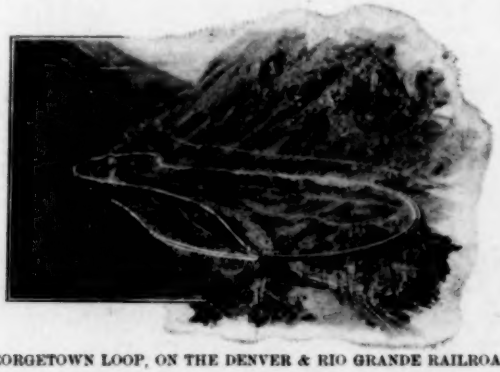
I wish you particularly to note that all of the phenomena which you have seen in connection with low temperatures are similar to those open to observation on various substances at ordinary ranges of temperature. In fact, there is nothing contrary to ordinary experience in any of the properties of matter in this interesting field. When first asked to speak to you upon this topic I was strongly tempted to decline because of the sensationalism which has been developed in connection with this topic as well as many others in physical science. A few years ago the unscientific public and many supposed scientific men were greatly enthused by lyceum bureau lecturers on this wonderful liquid, and magazine articles written by a promoter who succeeded in floating the stock of a concern for the manufacture of liquid air. The intense desire of the American public to make money by getting something for nothing was seized upon by the promoter to further his own ends who stated that when once started the liquid air was self producing and picturing in glowing terms the tremendous possibilities in the development of power. It is perfectly true that an enormous pressure may be produced by applying heat to liquid air in an enclosed space. But this gives no indication whatever concerning the energy relations involved and liquid air has not proved in any way an exception to the fact

that work must be performed in order to get returns. Not only this, but even conservative speculators were imbued with the idea of utilizing this exceedingly cold liquid as a refrigerator agent. A party not 1,000 miles from an institution where accurate information regarding the properties of this liquid might have been obtained, lost about $60,000.00 in efforts to utilize liquid air for refrigeration purposes. Note that a refrigerating temperature below zero degrees is seldom required in practice and that for this limit 80 lbs. of liquid air will be very approximately equivalent to 95 lbs. of ordinary ice. Under the conditions of operation in the plant where this air was obtained, the cost of production comes pretty close to $4.00 per liter, including breakage of expensive containers, interest and depreciation, attendance, etc. It is hardly to be expected that refrigerating plants utilizing this liquid would be financially successful.

It may naturally be asked, What is the use of liquid air and of low temperature investigations? As fas as the scientific man is concerned, increase of knowledge of nature pure and simple is a sufficient excuse for the expenditure of money and time to any extent. Even to those, who, unfortunately, look only at immediate commercial returns, we may still say that this liquid has proved to be of considerable use and to two cases I especially call your attention. In the preparation of oxygen for medicinal purposes the evaporation of liquid air gives a product much less likely to contain poisonous materials than that obtained by most chemical processes. Another application which may appeal very directly to some of you at some time of your life, is in the production of chloroform for use in anaesthesia. It has been found that the presence of impurities in chloroform add very greatly to danger to life in its use, and the only way in which perfectly pure chloroform can be practically produced is by freezing out the impurities by the use of liquid air as a refrigerating agent.

In conclusion I wish to urge upon you one and all as teachers to beware of sensationalism and *yellow* science. While maintaining an open mind regarding physical theories and admitting possible limitations of so called natural laws, you, of all mankind, should stand firmly for the general results of sound scientific experience and not eagerly seize each passing will-o'-the-wisp which the public, still influenced by superstition and mysticism, fondly hopes will, miraculously, enable them to violate natural laws with impunity. How often in the last few years have even reputable

magazines joyfully announced the final overthrow of that barrier to the realization of mystic hopes, the conservation of energy. Liquid air, self-regenerating, to drive our flying machines for nothing, radium overthrowing all the ponderous laws of physical science, and to be not only the inexhaustible storehouse of thermal energy but also the long sought fountain of youth and cure for most diseases. And not a little of this sensationalism is due to those who should be high priests at the altar of truth instead of demagogues selling their birthright for the miserable mess of pottage of popular excitement. The world has a right to expect that teachers and investigators in science should live up to the highest scientific ideals, and if the educational results of science do not always compare favorably with those of the subjects it has displaced, is it not largely due to our failure to realize its dignity and power? Do not imagine for a moment that I believe physical science should not be made attractive, or that striking experiments may not be legitimate and useful. Is it necessary to surround science with mysticism or to teach what the ripest results of experience contradict, in order to create a transient wonder and to keep up interest in our chosen subject? One who in any small degree comprehends the real wonder of our environment, the subtle beauties of the simplest natural process, visible to those, who, having eyes really see, can hardly fail to be an attractive teacher, even though building no air castles to obscure the temple of truth. To develop sound appreciation of science, to increase its usefulness in daily life, to make it satisfy the highest intellectual cravings and, finally, to dispel the idea of a vacillating and uncertain mystic ruling power whose laws may be circumvented at will, is the high calling of each and every real teacher of science.



GEORGETOWN LOOP, ON THE DENVER & RIO GRANDE RAILROAD

REFERENCE LIBRARY FOR CHEMISTRY.*

By B. W. Peet,
Michigan State Normal College.

The following list of books is submitted as a basis for a chemical library for high schools. The titles mentioned have been selected in the main with reference to the needs of the teacher, although a number are also adapted to the use of the pupils. The books have been classified and only a comparatively few are placed under each heading. It seemed advisable to limit the number, yet include the most of the best books published. The list has been submitted to several university professors and high school teachers of chemistry and has in general met their approval.

Beginning students can make little use of reference books until they have acquired some knowledge of the general principles of chemistry, so it is recommended that students be not asked to make any special study of reference books until the second half of the first year's work.

I. HIGH SCHOOL TEXTS.

Essentials of Chemistry, Hessler and Smith. 405 pages. Benj. H. Sanborn & Co., Chicago	$ 1 20
Descriptive Chemistry, Newell. 590 pages. D. C. Heath & Co.	1 20
Elementary Chemistry, Linebarger. Rand, McNally & Co.	1 00
Elementary Chemistry, Bradbury. 486 pages. D. Appleton & Co.	1 20
Modern Chemistry, Peters. 412 pages. Maynard, Merrill & Co.	1 00
Introduction to the Study of Chemistry, Remsen. (New edition 1906.) 518 pages. (Briefer Course.) Henry Holt & Co.	1 12
Elementary Inorganic Chemistry, Newth. 288 pages. Longmans, Green & Co.	90
Experimental Chemistry, Newell. 410 pages. D. C. Heath & Co.	1 10
Elementary Principles of Chemistry, Young. 358 pages. D. Appleton & Co.	1 20
Elementary Manual of Chemistry, Storer & Lindsay. 453 pages. American Book Co.	1 20

II. LARGE DESCRIPTIVE.

An Introduction to General Inorganic Chemistry, Alex. Smith. Pp. XVIII plus 780. Century Co.	$ 2 25
Outlines of Inorganic Chemistry, Gooch & Walker. 514 pages. Macmillan Co.	1 75
Text-Book of Inorganic Chemistry, Newth. 602 pages. Longmans, Green & Co.	1 75
Text-Book of Inorganic Chemistry, Holleman. 458 pages. John Wiley & Sons,	2 50
Inorganic Chemistry, Advanced Course, Remsen. 850 pages. Henry Holt & Co.	2 80
General Inorganic Chemistry, Freer. 559 pages. Allyn & Bacon, Boston	3 00
The Principles of Inorganic Chemistry, Ostwald. Translated by Alexander Findlay. 785 pages. Macmillan Co.	6 00
Treatise on Chemistry. Roscoe and Schorlemmer, Vols. I and II. (Inorganic.) D. Appleton & Co.	8 00

*Read before the Chemical Conference at Ann Arbor, Mich., March 29, 1906.

III. ANALYTICAL.

Qualitative Chemical Analysis, Prescott & Johnson. 420 pages. D. Van Nostrand Co., New York ..... $ 3 50
Qualitative Chemical Analysis, A. A. Noyes. 89 pages. Macmillan Co. 1 25
The Principles of Qualitative Analysis from the Standpoint of the Theory of Electrolytic Dissociation and the Law of Mass Action, Dr. Wilhelm Boettger, translated by Wm. G. Smeaton. P. Blakiston's Son & Co. .....
Analytical Chemistry: Qualitative Analysis. Treadwell. Translated by Hall. 466 pages. John Wiley & Sons. 3 00
Quantitative Chemical Analysis, Clowes and Coleman. 602 poges. Co., New York 4 00
Quantitative Chemical Analysis, Olsen. 513 pages. D. Van Nostrand P. Blakiston Co., Philadelphia 3 50
Analytical Chemistry. Quantitative Analysis. Treadwell. Translated by Wm. Hall. 654 pages 4 00

IV. DICTIONARIES.

Dictionary of Chemistry. Watts. 4 vols. Longmans, Green & Co... $65 00
A Dictionary of Chemical Solubilities, Comey. 515 pages. Macmillan Co. 5 00
The Chemists' Pocket Manual. Chemical Publishing Co., Easton, Pa. 2 00

V. THEORETICAL AND PHYSICAL.

Introduction to Physical Chemistry, Walker. 332 pages. Macmillan Co. ..... $ 2 50
The Elements of Physical Chemistry, Jones. Macmillan Co. 4 00
Chemical Theory for Beginners, Dobbin & Walker. 236 pages. Macmillan Co. 70
Physical Chemistry for Beginners, VanDeventer. Translated by Boltwood. 154 pages. John Wiley & Sons, New York 1 50
Theoretical Chemistry from Standpoint of Avogardo's Rule, Nernst. Macmillan Co. 5 00
Elements of Physical Chemistry, Morgan. 299 pages. John Wiley & Sons 2 00

VI. HISTORICAL.

History of Chemistry, Von Meyer. Translated by McGowan. 544 pages. Macmillan Co. ..... $ 4 50
History of Chemistry, Venable. 172 pages. D. C. Heath & Co. 1 00
Heroes of Science-Chemists, Muir. 350 pages. Thomas Nelson & Son 1 50
History of Chemistry, Ladenburg. Translated by Dobbin. 375 pages. University of Chicago Press 1 75
Essays in Historical Chemistry, Thorpe. 381 pages. Macmillan Co. 2 25

VII. ORGANIC.

Organic Chemistry, Perkin & Kipping. 552 pages (2 vols.). J. D. Lippincott Co., Philadelphia ..... $ 2 00
Organic Chemistry, Remsen. 364 pages. D. C. Heath & Co. 1 30
Organic Chemistry, Richter-Smith. 2 vols. P. Blakiston Co., Philadelphia 6 00

VIII. MISCELLANEOUS.

Outlines of Industrial Chemistry, F. H. Thorp. 528 pages. Macmillan Co. ..... $ 3 30
Methods of Glass Blowing, Shenstone. 96 pages. Longmans, Green & Co. 50
Chemistry of Daily Life. Lasser-Cohn. J. B. Lippincott Co 1 75
The Teaching of Chemistry and Physics, Smith & Hall. 377 pages. Longmans, Green & Co. 1 50

Chemical Lecture Experiments, Benedict. 476 pages. Macmillan...	2 00
Conversations on Chemistry, Part I. Ostwald. Translated by Elizabeth Ramsey. John Wiley and Sons. 250 pages.....	1 50
Conversations on Chemistry, Part II. Translated by S. K. Turnbull. 373 pages. John Wiley and Sons.....	1 50
Chemistry of Plant and Animal Life, Snyder. 406 pages. Macmillan Co.	1 40
Story of a Piece of Coal, Martin. 165 pages. D. Appleton & Co....	40

IX. PERIODICALS.

School Science, 440 Kenwood Terrace, Chicago. Monthly, October to June, inclusive.....$	2 00 per year
Journal of the American Chemical Society, Eschenbach Printing Co., Easton, Pa. Monthly.....	6 00 per year
Science, 41 North Queen St., Lancaster, Pa., or 66 Fifth Avenue, New York. Weekly	5 00 per year
Science Abstracts	
Engineering Index	
Scientific American	

DISCUSSION OF LIST.

A few of the recently published high school texts are very valuable reference books for students to use along with their own text, and they can usually be obtained for such purposes gratis of the publishers.

LARGE DESCRIPTIVE.

The most useful and extensive work of reference is the inorganic portion of Roscoe and Schorlemmer's Treatise on Chemistry. It deals with the historical, descriptive and theoretical parts of nearly all chemical subjects. The first two volumes have recently been revised.

The other books mentioned under "Large Descriptive" are university texts. Newth is noted for its attention to industries, Holleman for the interweaving of the Electrolytic Dissociation Theory with the descriptive and theoretical part in a simple way, Remsen and Freer for lucidity and discussion of general relations. The tables of comparisons of the elements in the various families and the discussion of the periodic law are well represented in Freer. Ostwald's Principles of Inorganic Chemistry is a modern classic and should be in the hands of every teacher who can afford it. The new notions and theories of scientific chemistry are very cleverly presented. The author has taken particular care in the development of the conception of ions. It is a text-book of *pure* chemistry, so little attention is given to the related sciences and arts.

Every teacher of chemistry should have access to Alexander Smith's Introduction to General Inorganic Chemistry* (just

*See the review of this work, under "Book Reviews," in this number of SCHOOL SCIENCE AND MATHEMATICS.

published). It is full of good ideas and methods of teaching different subjects. The chapters on Solution, Chemical Equilibrium, Dissociation in Solution, Electrolysis and Chemical Behavior of Ionic Substances are intensely interesting and well written. The theoretical side of chemistry is as clearly presented here as in any book on the market and it is up to date. The book contains about eighty pages of matter in smaller type which is intended especially for teachers and contains discussions of many points upon which teachers find difficulty in getting information, and also discussion of subjects upon which teachers are apt to go astray.

ANALYTICAL.

Prescott & Johnson's book is noted for the number of balanced equations and explanations of chemical reactions. Treadwell's Qualitative Analysis is a recent book and is generally considered an excellent one. The descriptions of the operations are full and clear, the equations are very fully treated and the treatment of acids is unusually extensive. The author utilizes the theory of electrolytic dissociation, mass action and hydrolysis, and explains their relations to analysis. Fresenius was long considered the best text-book on Qualitative Analysis, but it is not now generally thought of as up to date so it is not included on the list. One of the most recent books on qualitative analysis is Boettger's, now being translated by Wm. G. Smeaton of the University of Michigan. It makes practical application of the dissociation theory in the study of the metals.

If one wishes a book on quantitative analysis that describes methods of manipulation in detail and at the same time includes a compilation of the best methods of analysis, both Olsen and Clowes and Coleman are highly recommended. Olsen is more recent and seems to be the favorite in this country so far as it is known. In the back part of the book are chapters on water, gas, coal, iron and steel analysis.

DICTIONARIES.

Watt's dictionary of four volumes contains articles varying in length from a few lines to several pages, on nearly every chemical substance and every topic in scientific chemistry. It is often convenient to have a reference book on the solubility of chemical compounds to check results obtained by experiments in the laboratory or to aid one in making up solutions. Comey's dictionary

is recommended. Meade's Pocket Manual contains many valuable tables and much general information.

THEORETICAL AND PHYSICAL.

The books on Theoretical and Physical Chemistry are recommended for the use of the teachers of chemistry only. All that is necessary for students' use can be found in any good college text-book such as Alexander Smith's or Holleman's. Walker's Physical Chemistry is clear and more easily understood than most of the books on the market. The revised edition of Morgan is also very readable and a general favorite. Dobbin and Walker and VanDeventor are elementary. Jones is more complete, not so easy to comprehend but a very valuable reference book.

HISTORICAL.

No chemical library would be complete without some book on historical chemistry. A fund of information can be obtained here for making the every day lessons interesting. The teacher as well as the pupil will get a more comprehensive view of chemistry by knowing the struggles and failures and triumphs of the men who have made the science for us.

A very satisfactory book is Ladenburg's, translated by Dobbin. It is a compilation of a course of lectures and is presented in an interesting style. If one wishes a more complete treatise, Von Meyer is excellent. A very brief but reliable history of chemistry is Venable's. Heroes of Science by Muir, and Essays in Historical Chemistry by Thorpe, take up only a few subjects but they both read like romance and the pupil, as well as the teacher, will read them with delight.

ORGANIC.

A very satisfactory reference book on Organic Chemistry is Perkin & Kipping's. Remsen is very good but it is often so brief that parts are not easily understood. Remsen is desirable, however, because it is so full of easy experiments to illustrate many subjects. A very comprehensive view of the carbon compounds is to be found in Richter-Smith's book. The student and the teacher, with little preparation in organic chemistry, will probably get the most satisfaction from Perkin and Kipping's two volumes.

MISCELLANEOUS.

The industrical or practical side of chemistry always appeals to the average student. Thorpe's book seems to be a general

favorite. It requires but little encouragement to make the pupils anxious to read about the great chemical industries, such as the manufacture of soap, cooking soda, sulphuric acid and the metallurgy of iron. This book should be in every chemical library.

The Chemistry of Daily Life by Lasser-Cohn and The Chemistry of Plant and Animal Life by Snyder are intensely interesting and practical. The latter book deals with the chemistry of digestion, of foods, and the rational feeding of animals, including man.

Every wide awake teacher should have access to the teaching of Chemistry and Physics by Smith and Hall. It is full of suggestions on instruction in the class room and laboratory, tells how to present many difficult subjects and discusses the equipment of laboratories and lecture room.

Conversations on Chemistry, Volumes 1 and 2, by Ostwald are written in the form of a dialogue between the master and pupil. The teacher will get valuable ideas of presenting many difficult subjects to his students by reading these books. He is taught to realize the hard things from the pupils' standpoint.

It will add much interest to the development of any subject to illustrate it with other experiments than the ones given in the text or laboratory manual. Some such reference book as Benedict's Chemical Lecture Experiments is very desirable. It is full of interesting experiments on most topics in inorganic chemistry.

The best way to keep in touch with what is going on in the chemical world is to read at least one journal. The journal of the American Chemical Society is highly recommended because of the review given in each number of the leading articles appearing in different journals and also the mention and review of the books published on chemistry.

SCHOOL SCIENCE AND MATHEMATICS contains articles on all branches of science taught in the public schools, including mathematics. It is a magazine especially for the teachers of high schools, and no high school library should be without it. In addition to valuable and very practical articles written by university professors and high school teachers of prominence, brief reports of the science meetings in various parts of the United States are given.

The busy science teacher is too often satisfied to confine himself to the texts in use and pays little attention to what is being done in the scientific world. No progressive teacher, no matter what his salary may be, can afford to be without access to some science journal. It gives him inspiration to do better work, it keeps his mind more active, it helps more than any one thing to keep him out of a rut. It is not so important what journal the teacher reads, but read some good one and keep in touch with the scientific spirit of the time.

In conclusion let me emphasize that the teacher of chemistry cannot do his duty unless he has access constantly to at least one good reference book in each of the main branches into which the science is divided, namely: inorganic, organic, analytical, historical, physical, and industrial, and also reads some good science journal.

WHAT PHYSICS ASKS OF MATHEMATICS.

By Chas. H. Slater,

McKinley High School, St. Louis, Mo.

Physics teachers are almost daily confronted with the fact that their pupils find difficulty in handling the mathematical part of the subject. (1) Mathematical interpretation and statement of the variously related data of physics problems proves a stone wall to most of our pupils; although in algebra they have stated many equations the solution of which have given A's and B's ages, or the time when the hands of the clock will again coincide, How old is Ann, etc.

(2) Principles learned and applied with facility in the solution of these problems and equations of the mathematical textbooks are beyond the power of the pupil to apply in this new field.

(3) Computation is labored and beset with errors. Many simple operations requiring repeated explanations notwithstanding the training in arithmetic.

(4) In geometry, lines, surfaces, angles, etc., are studied constantly and their relations thoroughly investigated, but not a pupil enters the physics laboratory with any idea of accurate measurements of these familiar things, nor has he first hand knowledge that these actually demonstrated facts are true in practice.

The purpose of this paper is to urge a closer relation between mathematics and physics, and to indicate some of the ways in which the product of the mathematics instruction may be of more use to the pupil in his other work. But before answering the above question directly let us examine a few of the concrete conditions which the correlation of these subjects suggests.

Mathematics solves the problems of physics but how many text-books on mathematics reveal any practical use to which it is put other than in preparing to pass examinations? How many pupils see any application of mathematical principles to the problems in which they are applied? Our real educational growth is said to be from concrete to abstract. Our pupils have abstract mathematics first, and later in physics the concrete. They see little relation between the two and consequently are not interested in the subject except in so far as they are entertained by striking experiments and qualitative explanations. Hence we often hear the remark from teachers and pupils that mathematics is unpopular and that it also makes physics unpopular, both in text and laboratory work. But shall we then hasten to give courses in physics with little or no mathematics?

Technical grammar, the mathematics of English, made this subject difficult and unpopular. Now, they are so correlated that English is a most interesting subject. Nor is grammar neglected, it is necessary to an understanding of the language. It is a method of study or investigation and not an end in itself. In a very similar way mathematics is a method of study of science. It is necessary to anything more than a superficial knowledge of the subject.

Popular knowledge and reading abounds in references to candle power, amperes, velocities, temperature, horse power, machines, lenses, mirrors, etc., an understanding of which is most meager without some mathematics. We should remember that mathematics had its origin in trying to solve practical problems. Mathematical knowledge has grown because it has been useful. More persons are using mathematics and science today than when most of our present texts in these subjects were first written, for example, engineers, electricians, surveyors, navigators, chemists, architects, mechanics, agriculturists, contractors, masons, carpenters, manufacturers, students, accountants, musicians, etc. Physics must be quantitative to meet the needs of these professions; in other words it must be mathematical.

Our boys and girls then should study physics from the mathematical point of view, but how can this be done with a mind prepared only along certain abstract lines? Little wonder then, they fail to solve the simplest problems and equations of physics. It has been suggested that the mathematics work be so supplemented that the application of the mathematics be given along with the subject itself. This will not only prepare the mind for the science work but react beneficially upon the mathematics. Correlation must necessarily be brought about by supplementing the text-book work. Can we not have more of the concrete brought in from physics, chemistry, and every day life, and have the mathematics applied in useful lines while being taught?

The conditions mentioned in the arraignment of the present product of mathematics mentioned at first, the position of the two subjects in the course with no preparation for them below the high school, the individual needs of the two subjects and the lack of unified action toward their correlation are conditions which seem to make it important that we have a free exchange of opinions along these lines, and if possible to unite on some concrete means of bringing these subjects into closer relation in closer relation. It is to this particular end that the following suggestive points of correlation are mentioned.

First, mathematics is asked to give the pupil the power of interpreting variously stated real relations instead of imaginary conditions which never occur. It is also urged that the pupil show considerable ability in translating the English to the mathematical language and *vice versa*.

Second, we ask further that the pupil in physics be especially helped by having a good working knowledge of a few particular topics of mathematics, which find especial and frequent application in physics; as (1) the correct manipulation of an equation to find any part of it, the other parts being known; (2) graphs; (3) ratio; (4) direct and inverse variation; (5) proportionality; (6) geometrical construction needed in the composition of forces and velocities and in geometrical optics; (7) trigonometric functions; (8) exercises or experiments which involve problems in mensuration.

Third, physics asks also that mathematics introduce frequent exercises involving accurate methods of construction and measurement, so that the pupil will come to the laboratory with ideas

of accurate measurements, drawing to scale, a working knowledge of the metric system, and frequent practice in computation.

The first of these three points of correlation, the power of mathematical interpretation and translation has been strongly urged by many teachers of both subjects. We should however go farther and introduce new and vial problems for the artificial lifeless type found in the present texts with few exceptions. In the most recent algebra I find: "Five years ago a father was 6 times as old as his son, and five years hence the father fill lack 5 years of being 3 times as old as his son will be at that time. What are their ages?" Also, A, B, C, and D have between them $1,025, B has twice as much as A, C has $50 less than B, and D has as much as B and C together. How much has each?" etc.

Contrast with these the following: "A and B run races of 300 yards; in the first race A gives B a start of 60 yards, and wins by 10 seconds. In the second race A gives B a start of 15 seconds and wins by 40 yards. What are their speeds?" Also, "Assuming the velocity of sound to be 1100 feet a second, find the distance of the point of discharge if 24 seconds elapse between seeing the lightning and hearing the thunder."

But someone says that physics problems cannot be understood. They will be when explained, as must also be done with each type of the old style problems. Practice problems are needed, but give us also many common sense, concrete problems. The pupil will not find it so new and strange when he encounters such problems in later studies and his mathematics will cease to be dry and meaningless, since they will seem to lead to something useful. New live problems are needed. Many mathematics teachers have taught physics, and *vice versa*. Can we not, by working together, make a suitable selection of problems to be used in supplementing the text as above indicated?

In regard to giving special attention to particular mathematical topics for the benefit of physics, probably the most important is (1) that relative to a correct manipulation of the equation, regardless of letters used and which are known and unknown. When x, y, z, a, b, c, are involved less difficulty occurs, but s, m, d, v, t, f, a, etc., cause all kinds of trouble. Nearly every term, algebraic expression and equation used in physics should be found in the problems of algebra. Every law of physics may be expressed as an equation and thus become part of the work. There

are but few algebras in which these will be found. Pupils should know that these letters represent certain concrete things and that equations represent relations which will be studied later. When they can be handled easily, many physical problems may be introduced to be solved by substitution of known values. In this way the pupil sees some application of algebra and at the same time becomes familiar with the terms of physics.

(2) Of the special topics mentioned probably the one receiving most attention today is that of graphs. They are useful in many ways, first to the mathematics teaching in giving a better understanding of linear, simultaneous and quadratic equations. Graphs enable one to actually see, and to the pupil "seeing is believing." The equations, their properties, and solutions now become real and concrete—they are no longer a combination of signs and symbols for the artful juggler. The number and kind of solutions possible are more clearly shown and understood with the graph than by theoretical proofs, which are more bewildering than helpful to the average pupil. Secondly, experimental results in physics may very often be expressed in the form of a graph. John Perry says, "the basis of all applications of mathematics in physics and engineering is the fact that *any physical phenomenon* which is directional, such as a force, a velocity, an acceleration, a stress, the flow of a liquid, etc., may be represented in a most perfect manner by a straight line." Many other relations are represented by curved lines, as in Boyle's Law, capillarity, the pendulum, elasticity, polarization of a voltaic cell, etc. The pupils can plot curves from actual physical data which will give them such lines as those indicated in addition to plotting $x-3y=9$, $xy=12$, etc. The squared paper used for this work may be used in other ways also, as drawing to scale, which is very important in itself.

Three other closely related subjects of mathematics which fall far short of usefulness in physics, if this be in any measure the test, are ratio, proportionality and variation. They have been studied in arithmetic, algebra, and geometry, but to no avail as far as physics can judge. A clear understanding of most laws of physics is reached often by many comparisons of data, which is done by finding the ratios of the measured magnitudes. The ratios determined, the law is stated by indicating how one quantity varies with the other, directly or inversely, etc. Many problems illustrating the physical laws are most easily solved when

based upon the idea of proportion. But for the most part pupils can do nothing with the application of these to real problems. They remember only that "the product of the means equals the product of the extremes." Why not omit most of the theoretical discussion of these topics and spend the time on some familiar and workable problems such as are suggested by the report of the committee on the correlation of mathematics and physics in the secondary schools, which should be in the hands of every teacher of these subjects.

It is generally true that the teacher of physics is obliged to teach whatever geometry is needed for the subject. The pupil very soon encounters the composition of velocities and forces in the text and soon again in the laboratory. He has had three of four weeks in *demonstrative* geometry and hence must "be shown" how to complete the parallelogram to find the resultant of the component forces. Reflected motion, stability, curvilinear motion, pendulum, lever, inclined plane, wave motion, images in mirrors and lenses, and index of refraction of light are topics requiring a workable knowledge of constructional geometry.

Since our courses in geometry and physics in some schools begin at the same time, could not the first three or four weeks be given to constructional work in lines, angles, perpendiculars, parallels, triangles, parallelograms and circles, etc., using cross section paper, protractor, dividers, diagonal scale and the metric rules? This would give the necessary preparation for the first part of physics—the other topics would be studied in time if taken in the usual order This constructional work would also materially aid the mathematics. For 50 years it has been urged that inventional or constructional geometry should precede the demonstrative. It places the concrete before the abstract. In most schools, it is true, physics follows geometry and so gets the benefit of this work being more thoroughly covered, but that is impossible when both are commenced at the same time. Hence the special necessity of early correlation of these subjects.

It would further be an aid to physics if the trigonmetric functions could be taught, especially the sine and tangent of an angle. Does any one see any objection to teaching these following the study of similar triangles? A knowledge of these is needed for electricity and light in the text and laboratory work.

Mathematics may also relieve physics of a few of its prelimi-

nary experiments in simple measurements, viz., the English and Metric measurements, determination of the ratio $C \div D$, use of the diagonal scale and graphical representation of results. But these would have already have been covered, if the work in graphs and constructional geometry be given as above mentioned. With special attention on the part of the mathematics and physics teachers to these eight points for special correlation and an endeavor to give practical exercises illustrating and supplementing the regular work, a live interest should be developed which would make it possible for the pupil to undertake physics with better results and more pleasure to himself from the mathematical point of view.

Thus far this discussion of what physics asks of mathematics has related primarily to the text-book side. There is yet what seems to me a third equally important point of correlation. What the first two requests, if granted, would do for the text-book work in physics, this third would very certainly do for the laboratory side of the work. As stated in the beginning, pupils enter the laboratory with no idea of accurate measurement and with little ability in correct computation. To the end therefore that the physics laboratory work might begin with the purely physical experiments and that computation might not be so labored, physics asks last that supplementary exercises involving accurate methods of construction and measurements be given frequently throughout the first two years in the high school. This will be quite as valuable to the mathematics as to the physics.

May we not introduce the use of the protractor, dividers, English and Metric scale very early, and a little before the close of the first year the use of the diagonal scale? These may then be used in accurate measurements to solve an exercise, or to verify a certain theoretical statement before it has been demonstrated. When the pupil has verified the fact by his own measurements, he will soon be convinced that the conclusions of the demonstrated fact are sure also. Since the exercise is to be constructed and measured accurately, and computations made therefrom it has a three-fold aspect—it is not mere arithmetic, but in it is geometry, drawing, computations, all depending on the pupil's care and knowledge which are thus tested.

In the construction of triangles, this exercise would illustrate a suitable type. Find the area of a triangle whose base is 4 inches, and its adjacent angles are 60 and 75 degrees. The con-

struction should show all auxilliary lines and the measurements and results should be carefully tabulated.

In conclusion I would say that this view of the correlation of these two subjects is a middle view. There is the orthodox pure mathematician on the one hand, and on the other the enthusiast who would teach algebra, geometry, and physics all at the same time practically. There are very few teachers who could carry these three subjects together successfully. In this the pupil does many physics experiments for his mathematics. The two subjects are dovetailed into each and consequently would lose much of their individuality which would be undesirable especially for the beginner. It would seem to be confusing to the pupil. It could not be done without present course of study and it is more revolutionary than evolutionary.

The middle view as outlined in no way conflicts with the course of study, can be easily handled by most teachers, many have been teachers of physics, since it consists of work wholly of a supplementary nature, and by being concrete it will appeal to more pupils in mathematics. Last and most important to us, it will make it possible to do more and better work in physics.

PARAFFINED WIRE POTS FOR SOIL CULTURES.

By B. E. Livingston.

It is suggested that in case of experiments with potted plants, the porosity of the earthen pots in use in general for such experiments is such as to present conditions quite unlike those existing in the earth. By dipping wire baskets in paraffin until the wire becomes well coated a container is produced which presents conditions more nearly in accord with normal conditions. Experiments with plants grown in such baskets show that the roots do not become massed against the wall of the pot, as is usual, but are within the earth.—*Plant World, March, 1906.*

THE HELIODON.*

By Jos. F. Morse,
Medill High School, Chicago.

I. DESCRIPTION.

The Heliodon* is essentially an adjustable, three-dimension diagram for showing the path taken by the sun through an observer's sky at any latitude, in mid-summer, mid-winter, and at equinox.

It consists of a disc, representing the horizon circle of the observer, encircled by three sun-path rings. The middle ring, pivoted to the east and west line of the disc, represents the sky-path of the equinox sun, as viewed from center of disc. The outer rings—fixed parallel with the middle ring at a distance of 23½°, as measured on the horizon disc—show the solstice sun-paths as seen from center of disc, the northward ring representing the path of the June sun, and the southward ring of the December sun.

FIG 1.

The rings are vertical to the disc when adjusted for the equator. For northern latitudes they are leaned southward, and for southern latitudes northward from the vertical position, as many degrees in every case as the latitude in question is from the equator. The amount of inclination is gauged from the middle ring and measured by a quadrant of the same radius as the disc.

When adjusted for a given latitude, the Heliodon demonstrates the exact sky-path of the sun—direction from the observer throughout the day—at the equinox and solstice seasons, and the approximate sun-path at other times of the year.

* The Heliodon is manufactured by the Central Scientific Company, Chicago.

By showing the length of day in mid-summer and mid-winter and indicating the limits within which the sun shifts its rising and setting points and noon position, the Heliodon furnishes all the data needed for determining the kind of temperature seasons —so far as dependent upon the sun—that would be experienced at any latitude.

Thus, when the rings are vertical to the disc, the equator position, (see Fig. 2), the sun-paths are all bisected by the horizon plane, indicating a twelve-hour day the year through. In this position the noon sun is seen to be overhead at equinox, and to shift 23½° north and south of the zenith at solstice times. As seen from the equator, the June solstice sun rises and sets 23½° north of east and west, and the December sun 23½° to the south.

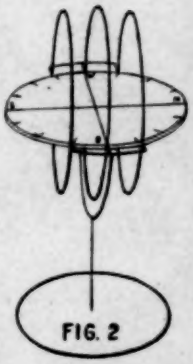


FIG. 2

As the rings are leaned southward for northern latitudes the June solstice day is lengthened and the December solstice day shortened at an equal rate; while the northward leaning of the rings for southern latitudes lengthens the December and shortens the June days—showing reversal of seasons on opposite sides of the equator. The length of solstice days (and nights) is read from the intersection of the time marks on the solstice rings with the horizon plane. (In the instrument, the true horizon plane is marked by a line bisecting the edge of the horizon disc.) The points of attachment of the rings represent six o'clock, morning and evening. The wires connecting the rings midway between points of attachment represent portions of the observer's meridian, and indicate noon and midnight.

As the rings are leaned away from the vertical, or equator position, in either direction, the rising and setting points of the solstice sun are thrown farther and farther from the east and west, until, in adjustment for the polar circles, the rising and setting sun meet on the north and south points of the horizon, giving a 24-hour solstice day and night (see Fig 3).

FIG. 3

Beyond the polar circles the solstice rings clear the horizon disc by a greater and greater space, showing an increasing period of con-

tinuous sunshine in summer and night in winter. In position for the poles the rings are parallel with the horizon, 23½° above and below it, indicating a six-months "day" and night (see Fig. 4).

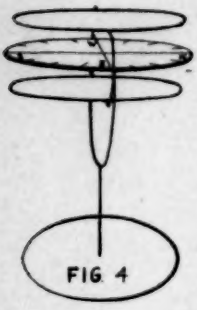


FIG 4

By showing the path of the sun below the horizon—from sunset to sunrise—the Heliodon demonstrates the position of the midnight sun and the lengthening of the twilight periods with approach to the poles.

Since the space between the solstice rings is bisected by the horizon plane in every position of the rings, all latitudes are seen to have six months of sunshine and six months of night in the course of a year—the long nights of winter just off-setting the long days of summer. The decreasing intensity of insolation with approach to the poles is thus seen to be due entirely to the increasing slant of the sun-paths, and not to any difference in the number of hours of sunshine received in a year.

It is also evident from the Heliodon that the increasing slant of the sun-paths for higher latitudes throws the sunshine more into one half of the year and the night into the other—causing a greater yearly range of temperature, as well as a lower average.

By adjusting the Heliodon for any given length of mid-summer or mid-winter day, and then measuring the inclination of the middle ring by the quadrant, the latitude required for any length of solstice day can be ascertained, without trigonometrical calculation.

The position of the sun's sky-path for other times of the year than equinox and solstice can be located approximately by imagining a ring placed parellel with the given rings at the proper interval between them for the time of the year in question—the given rings being three months apart. The yearly movement of the sun can be pictured by imagining one of the solstice rings moved to the position of the other and back, kept parallel throughout with its original position. In reality the sun moves from one solstice position to the other in a fine-threaded spiral.

The reason why the sun-paths vary with latitude, in the manner shown by the Heliodon, is best demonstrated to pupils by pinning a small pasteboard disc, crossed by north-to-south and east-to-west lines, to a globe at the different latitudes considered,

and rotating the globe, in right position to the sunlight (or artificial light) for equinox and the two solstice times. The approximate length of day is shown by the amount of rotation required to pass the disc through the sunlight. The direction of sunrise and sunset is opposite to that taken by the pin's shadow when first and last seen on the disc in the course of the globe's rotation. The varying height of the noon sun appears from the varying length of the pin's shadow in the noon position of the disc.

With immature pupils the shifting of the sun-paths with latitude, and its effect upon seasons may be studied out from the Heliodon without attempt at astronomical explanation.

II. A LABORATORY EXERCISE.

Aim: to show the effect of latitude upon the average intensity and seasonal distribution of the sun's heat.

Method: to study out from the sun-paths as given by the Heliodon, the kind of heat seasons that would be experienced at the equator, the north pole, and Chicago; and to summarize the results in temperature curves, or "hills."

It is assumed that pupils undertaking this exercise understand fully (1) that daily insolation is a resultant of two factors—length of day and height of noon sun; (2) that slanting sunshine is weaker in proportion to the increased area covered by a given beam of light, and that this area does not increase rapidly until after a considerable slant is attained.

(a) *Insolation seasons at the equator.*

Adjust Heliodon for equator as shown in Fig 2. See that pupils understands that in the course of a year the sun shifts slowly from one solstice position to the other and back again. Guide the study of the pupils by the following questions:

At what times of the year would the equator receive most heat from the sun? Why? Would there be cooler times? If so, when? Would the average heat (total heat received in a year) be much or little? Reason for answer. Would there be much or little variation of heat during the year? Explain answer.

When pupils have reached approximately correct conclusions, they may be shown how to picture the heat seasons of the equator to the eye by a temperature curve, showing two low "hills"—March and September—above, and two shallow troughs—June and December—below a line representing the average level of

heat—about 80°. (See equator curve E in accompanying diagram.)

(b) Insolation seasons at the North Pole.

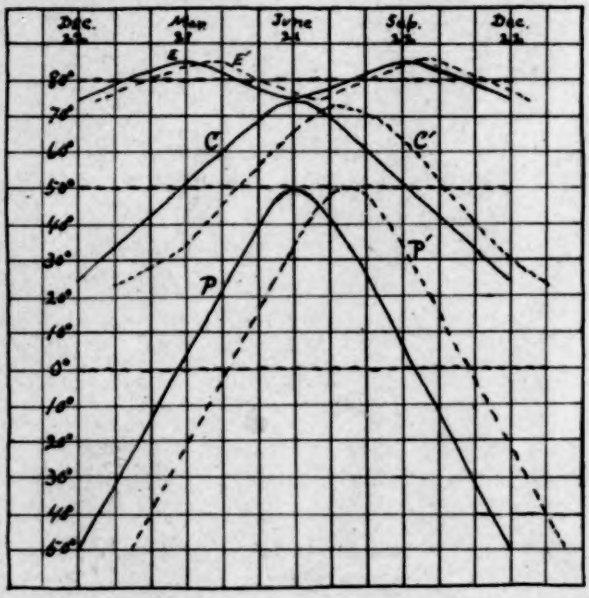
Adjust Heliodon for north pole (Fig. 4). See that pupils notice that in making the adjustment the June ring is thrown above and the December ring below the horizon.

Why do polar regions average so much colder than the equator? Do you think snow ever melts at the north pole? Give reason for answer. When would it be warmest and coldest at the north pole? Why? Would the difference between the warmest and coldest times be much or little? Explain. Would a temperature curve for the north pole show one, or two, hills? Draw a curve to show your idea of the heat seasons at the north pole. (An average of 0° and a yearly range of 100° may be given as approximately correct for the north pole. See curve P in diagram.)

(c) Insolation Seasons at Chicago.

The adjustment of the Heliodon for Chicago is nearly that shown in Fig. 1.

Why does Chicago average warmer than the north pole? Give two reasons why June is warmer than December at Chicago. How would the Chicago annual range of temperature—difference between the warmest and coldest months—compare with



that of the north pole? with that of the equator? Explain answers. On the same "checkerboard" with the equatorial and polar curves draw a yearly temperature curve for Chicago, with 50° for the average and 50° for the range. (See curve C in diagram.)

So far pupils have been allowed to assume that the warmest time of the year would be coincident with the highest sun and longest days, and the coldest time with the lowest sun and shortest days. To set them right in this matter, let the pupils construct—on the same checkerboard, but with a broken or colored line—the yearly curve for Chicago from observed data, averaged for a long period of time, explaining that in a long term of years north and south wind interferences with the sun's work will about balance up and cancel out, leaving approximate "sun-weather." The following monthly averages are for a 17-year period, 1874-1890:

Jan.—24°	Apr.—46°	July—72°	Oct.—52°
Feb.—28°	May—57°	Aug.—71°	Nov.—40°
Mar.—34°	June—66°	Sep.—64°	Dec.—30°

Inspection of the Chicago curve constructed from actual temperatures (see curve C′ in diagram) shows: (1)that July, not June, is the warmest month, with August a close second; (2) that September is 30° warmer than March. That this apparent discrepancy between theory and fact is due to the gradual accumulation of heat during one part of the year and gradual loss of heat during the other, may be brought out by class discussion.

After remodeling his theory regarding times of maximum and minimum temperatures at Chicago, to fit the facts, the pupil will readily see that essentially the same reasoning applies to polar insolation, and that the polar hill must be moved over a month or six weeks to the right. (See P′ curve in diagram.)

Equator heat seasons follow the sun more closely than those of Chicago or the north pole because there is so much less contrast at the equator between the cooler and warmer times of the year. Hence the actual equator curve is but a little to the right of the assumed curve first made. (See E′ curve.)

To summarize the results of their study the pupils may finally be given the statement that increase of latitude decreases the average intensity of insolation, by making the sun-paths more slanting, and increases the yearly range of temperature, by throwing more of the sunshine around one solstice and of the night around the other.

THE NATURE OF APPLIED PROBLEMS IN ALGEBRA.*

By David Eugene Smith, LL.D.,

Professor of Mathematics in Teachers College, Columbia University, New York.

TWO SCHOOLS OF MATHEMATICS.

From a time so remote as to deserve being designated as immemorial there have been two distinct schools of mathematicians. The first in point of time and numbers, though not of influence, is the school of practical mathematicians, the men who made our commercial arithmetic, settled our methods of computing, furnished our genuine problems in mensuration and mechanics, and used the mathematics of the theorists as far as they had need for them. The second school is that of the theorists, the dreamers in the science and the mystery of numbers, the logicians who built up our geometry, and the men who have posed or ranked, and now rank or pose, as original investigators. The smaller minds in each school have always affected to look down upon the minds of the other, but the larger minds have sought rather to appreciate to the full the thoughts and labors and aspirations and results of the masters in both fields. So Thales, who in times remote founded the science of geometry, was one of those who did not scorn to use that science for practical ends, and Archimedes of ancient Syracuse, and Heron of Alexandria, and Newton, Laplace, Lagrange, Kelvin, and Klein, are among the multitude of those who are imbued with the same all-embracing spirit.

In our day we are not free from the distinction, and the world will probably never break down the barrier. On the one side the inquiry will be raised, *Cui Bono?* What is mathematics good for? What are the genuine applications? And on the other side the dogma of mental discipline will be sounded forth from the minarets of the city, and the faithful will be admonished that utility is the aim of inferior intellects.

Now two such schools of thought cannot exist through the centuries without each having the support of truth. There is never continuity founded upon falsehood. Hence it comes that the world's best teachers have always sought the truth in each

*A paper read at the December, 1905, meeting of the New York State Science Teachers Association.

of these schools. They have recognized the value of pure mathematics *per se*, but they have also recognized the value of the science as it has measured up in one way or another the quantitative side of human life.

Let us see, for example, what has been done in our day for arithmetic that we may estimate more clearly the possibilities for the reform along the higher lines of algebra.

THE QUESTION IN RELATION TO ARITHMETIC.

Now in arithmetic no considerable number of people seriously think of banishing the drill work with abstract numbers. The ability to work rapidly and accurately, developed by centering the thought on the operation itself and not on the interpretation of a problem, is considered vital. It is here that students are the most deficient when they enter college, and there is a just complaint in business that this phase of work is unduly neglected. On the other hand no one would think of having no concrete problems in which the pupils should translate a question into the language of number, and answer it in that language. The chief dispute of late has been as to the nature of these concrete problems. There are several possibilities for them:

(1) They may be mere puzzles, with no reference to practical life. Such puzzles are not without their value. They often arouse an interest through their very absurdity, as in the 7 cats killing 7 rats in 7 minutes, or through their evident impracticability, as in the pipes filling the cistern. But no one would advocate more than a few such problems, and common sense would suggest scattering them so as to recognize their peculiar value of enlivening the game that is being played in a mathematical course.

(2) They may be obsolete business problems as well as modern ones, on the principle that the reasoning involved is quite as good with unreal as with real examples. This would, of course, be a sound enough argument if it were not for the fact that it is unnecessary to give false ideas of the business world about us. Since just as good reasoning can be obtained from genuine problems as from those that give a distorted idea of practical life, the latter become pernicious. Such are problems in equation of payments, partnership involving time, alligation, all of which have been banished from modern arithmetics.

(3) They may be genuine problems gleaned from the world

about us, types of what the child is to meet, examples with an interesting and valuable content, questions that relate to the country's resources, to the daily life of our people, to the interests of the living rather than of the dead. All modern writers on arithmetic seek such problems, and it is only a matter of patience to find them in large supply. Such problems are sometimes criticised, but these criticisms will not deter their growth and extensive use with all text-book makers and all modern teachers. They are here to stay.

THE QUESTION IN RELATION TO ALGEBRA.

Now how is it with our algebra? Are the conditions similar to those of arithmetic? Have we an inherited mass of problems as open to criticism as the conglomeration of arithmetical examples that have come down to us in considerable measure from the 15th century? In other words, are there types to be considered corresponding to those already mentioned in arithmetic?

(1) What about the abstract work? It is not in the province of this paper to elaborate this topic. But briefly it may be asserted as probably not open to serious objection that there must be the same mechanical efficiency here as in arithmetic, and this better comes from manipulating a large number of abstract functions than from solving a smaller number of concrete problems. Facility, certainty, accuracy in this work is indispensable to all higher analysis and applied mathematics. This must, therefore, stand about as now, save that we might better the sequence, merely eliminating absurd concatenations that are never met, and substituting forms that are common in later work.

(2) What shall be done with the puzzles? My hope is that they may be retained within reason, simply for the amusement and interest that attach to them. If their value in this line is recognized, they are not open to objection. No student considers very seriously a problem about finding a number which increased by a seventh of itself equals 19 (historically the oldest problem involving a linear equation that has come down to us). It is interesting, it furnishes some valuable exercise, and as a type it is unobjectionable if used in moderation.

(3) But what is the outlook for genuine problems? Can we regenerate algebra as we have regenerated arithmetic? Is there a field in which algebra is really used, and which at the same time

is one that the pupil is prepared to enter? Can we touch life with the equation, tell any of the story of humanity with the formula, and give to a function a genuine social content? If we cannot vitalize algebra would it not be better to reduce it in quantity, with the confession that its mere educational value does not warrant, and that advanced mathematics does not need, the expenditure of a year and a half of a pupil's time in the manipulation of abstract and seemingly useless functions?

Personally I am an optimist here as elsewhere. I believe that if we search out uses for algebra, aside from the uses of the subject in higher studies, we shall find them. While I am forced to admit that we do not have a large number now, I believe it is because we have hardly begun to look for them. Twenty years ago a genuine American problem in arithmetic was somewhat of a rarity; people said it was necessary to preserve the inheritance because we had nothing as good to substitute; but today, authors are finding plenty of genuine applications, and they find them simply because they look for them.

THE POSSIBILITIES FOR ALGEBRA.

Now what are the possible fields for algebra? In the first place there is the formula, the most natural of all introductions to algebra. Instead of having a pupil begin by manipulating a lot of meaningless functions, it is perfectly feasible to let him see at least one use of algebra by beginning with genuine formulas. Happily the field is here a fertile one. The artisan's journals furnish many illustrations, the business rules of arithmetic furnish others, and elementary science supplies still more. Opposed as I am to turning our classes in mathematics into classes in physics, I am still aware that every child entering the high school knows enough of natural laws to appreciate formulas on the lever and on uniform motion . Formulas embodying the most common cases in general percentage, in interest, and in discount are not only easily understood by children, but they illustrate one of the chief uses of the algebra of which they are beginning the study. Hence there is abundant material of this kind for the first steps in the new science, material that is interesting, that is valuable, that is related to human life and that is practical for the artisan, the merchant, and the business man generally. Once the interest in such formulas is aroused, this interest will help to hold the pupil to his work in the fundamental

operations with abstract functions. But I am not without hope that even in the latter field a line of applications will be found, just as we find this line in arithmetic on the one hand and calculus on the other.

The next field of applications that has proved fertile thus far is related to the negative number. Instead of approaching this subject from the abstract side, or with the help of the single illustration of debt and credit, we now have abundant material of which the teacher can take advantage. Chronology, latitude, longitude, temperature, assets, games, the numbering of east and west streets, opposed forces, and many other similar lines of practical applications make the negative number seem as real and usable today as the positive.

The linear equation with one unknown also offers abundant practical material. One has only to seek and he finds. Percentage problems are particularly valuable in showing the practical uses of the linear equations, although the field is by no means limited to this group of applications. I should therefore look with suspicion on any algebra which should appear today without a considerable number of honest business problems under this head, problems that are interesting, valuable, informational, and as full of mental discipline as the old inheritances that have come down to us through the middle ages.

In the study of ratios, with those particular subtopics known as proportion and variation, we have an abundance of applications that are of the best type of excellence; problems with a genuine content, but not involving technicalities beyond the understanding of the pupil.

When we come to linear equations with two unknowns, however, we are not so fortunate, and if we increase the number of unknowns to three or more we have practically no good applications in any of our books. As a result we find ourselves resorting to the mere puzzle, to some absurd hypothetical case, To be sure we can clothe the problem in concrete language, as in finding the temperature of which the number of degrees Fahrenheit is 5 times the number centigrade, but such problems do not seem genuine, and really are not so. About the best class of problems of this kind at present available is that relating to the mixing of metals or of solutions, but the field is soon exhausted.

In the matter of quadratics we are still more troubled to find

genuine applications. Physical questions, like the well known problem of the point of equal illumination of two lights and certain problems relating to moving bodies, help us a little. They usually fail, however, because they involve technicalities beyond the knowledge of the pupil. We therefore find ourselves today with practically no good applications in quadratics.

I have now laid before you the general issue. Let me recapitulate:

1. If we can secure genuine, vital problems of the day to illustrate the various chapters of algebra, we shall gain in the interest, and we shall at the same time enhance the practical value of the subject without detracting from the educational value. This has been done in arithmetic, and it may be hoped for in algebra.

2. In certain lines we can find plenty of practical problems, although text-book makers have not as yet availed themselves of this material. These lines are notably the study of the formula, probably the most practical part of all algebra, and the linear equation with one unknown which ranks a close second in the matter of usability. To these may be added the negative number, and the graph which is so closely connected with it. The algebraic graph, however, is not of so great practical importance as is generally assumed. There may also be added the study of ratio in its largest sense, and the complex number.

3. In the work in simultaneous equations and in quadratics we have brought together only a very small number of good practical applications. It is, therefore, to this field of work that I invite your particular attention. I do not believe that we should give up the matter and say, "It is hopeless," "There are no such applications," "What is the use of bothering with the matter so long as the examining bodies remain asleep to the issue?" Just as some of our best teachers have discovered of late a wide range of practical and easily understood formulas, notably in artisans' journals and handbooks; just as a number of illustrations of what was once so puzzling, the negative number, have been discovered; just as the linear equation has of late come to be used extensively in throwing light upon many involved problems of arithmetic, so I have faith that we shall yet find genuine applications for our simultaneous equations and quadratics. It is worth much for us to get together like this and face the issue. It is helpful to clear the atmosphere and see where we stand. We can never solve a problem until we know what it is, and if you

and I know the goal we may, at least by our sympathy, assist others in attaining it.

I know very well that this suggestion will stimulate some to be on the lookout for genuine applications; that it will have an almost equal value of making a larger number sympathetic with educational advance; but that it will be entirely understood by a small minority. For if it be said that this is an appeal for utilitarian mathematics as against cultural, I have lamentably failed in presenting my case. My appeal here, as in arithmetic, is simply this: Let us cast away pretence and sham in our applied problems; let us have them stand on their merits as puzzles, like the number guessing, as interesting inheritances like the cistern pipes and the couriers, or as real and usable applications taken from the commercial, the scientific, or the social life of today. Let us neglect no necessary abstract work; let us drill upon processes until they become mechanical; but let us make the subject more interesting by honest applications, more valuable by a study of real conditions, more inspiring by furnishing a key to new chambers in the quantitative structure through which we pass. Above all, let us not despair of doing all this, rather taking hope at the success in already finding applications not thought of only a few years ago, and stimulating by our sympathetic interest all who are laboring with honest purpose to improve our algebraic teaching, to rejuvenate this the third oldest of the mathematical sciences, and to make attractive a subject which has of late become rather the most gloomy of the antechambers through which the student passes to reach the vital, progressive, pure mathematics that lies beyond.

RESEARCH AND GRADUATE WORK FOR TEACHERS OF BIOLOGY IN HIGH SCHOOLS.*

By Esther F. Byrnes, Ph. D.
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As we are teachers who have gathered together to discuss various phases of our professional work, I think that the subject on which I have been asked to speak, "Research and Graduate Work for Teachers of Biology" must fall under two heads. First, the value of close contact with the subject matter of the science to the teacher himself; second, the influence of the teacher's relation to his subject, on those whom he instructs.

To those intimately acquainted with the science of biology, it is known to be, more than most subjects, a rapidly growing field of knowledge. Within the limits of this field, there are many lines of interest, each demanding its own methods of investigation and yielding results that though often illuminating, are as often varied and perplexing, and all are set forth in a cloak of terms so technical as to convey little meaning to the layman without interpretation. But one common interest underlies all of these many fields of effort, namely, the reduction of unexplained phenomena in the organic world—in a word, evolution.

In 1877, Prof. Huxley wrote, "Evolution is established; it is only the nature of the physiological factors to which that evolution is due which is still open to discussion." Let us examine this doctrine of evolution from the point of view of the contributions that have been made to the explanation of its methods within the past fifty years. In Prof. Huxley's day the great contribution was Darwin's Theory of Natural Selection; at first discredited as incompatible with accepted and enshrined beliefs, now universally accepted among those who care to be informed, as one of the factors in explaining organic evolution.

Following Darwin, the wonderful revelations made by the microscopic study of the cell grew into a body of knowledge that when formulated, found expression in Prof. Weissmann's "Continuity of the Germ Plasm" as a contribution to the solution of the method of evolution.

More recently the modern spirit of experimentation has dis-

*A paper read at the December, 1905, meeting of the New York State Science Teachers' Association.

covered a new approach to the solution of the method, and has also discovered that even its method is not new. To-day, this method is expressed in a single word "Mendelism," and one of the most modern phases of biological research is the attempt to correlate cell phenomena with the results of experimental studies in the rearing of animals and plants.

Pasteur disproved the old theory of spontaneous generation. To-day we have the science of Bacteriology with "radiobes" at one end of the series, and the bacteria of dairy products and the the whole field of germs in their relation to disease at the other.

About two years ago a new element, Radium was discovered; to-day its effects on organic life are being widely investigated. Without a modern knowledge of physiological chemistry certain avenues of biological investigation are closed to us. And what great contributions to our knowledge and even our conduct of life may we not expect as a result of verified observations in the field of experimental work in inheritance?

These few illustrations may suffice to show that the closest touch with the many and diverse fields of biological investigation is not only of value, but is an unquestioned necessity, if we are in any measure to keep pace with our intellectual opportunities. If there be any question as to whether research and graduate study be the essential method by which we are to come into our intellectual inheritance, let us ask what biological conceptions have most affected our thinking of today? out of what data have those been formulated? and how have those data been revealed?

One of the conceptions most concerning us as teachers is that on which the so-called Genetic Method of teaching is based, for it recognizes as a fundamental principle of psychology, that the child repeats in his own mental development, the development of his race. This is merely the application of the dictum—Phylogeny is indicated by Ontogeny, or, the history of the race is repeated in the individual:—a conception based on the comparative anatomy and embryology of many types.

The method of research by which all great generalizations have been verified, is the laborious method of repeated observation, with here some observations affirmative to theory, and there, some observations contradictory to it. The final conclusions must take both series of observations into account—must be the mean of both. What better instrument for the training of the

mind, for the tempering of the judgment, than the great store house of biological processes and phenomena?

One of the greatest needs of the teaching profession is a high ideal of scholarship. Pedagogy seeks to put teacher and pupil in right relations with each other. But I sometimes think it neglects the very heart of the matter in paying so little attention as it does to the conditions under which the result aimed at is to be secured; for teachers are given little opportunity to be progressive and well informed, and to nurture enthusiasms.

Probably nowhere in the curriculum of our educational system are such heavy demands made on the instructor as in the secondary schools; for while the demands for scholarship are high, the claims on the personality and on the time of the teacher are so great as to afford little opportunity for him to meet the higher scholastic requirements demanded of him; ultimately he becomes depleted both in mind and body.

The teacher needs some source of recovery and inspiration. It seems to me that teachers of biology might find this in the higher walks of their own calling; in research work, in some of the many fields open to them, for this is an avenue to far more than to mere scientific detail. So large a field as biology, offers an exceptional opportunity for the choice of a recreative pursuit as well as many opportunities of association with those whose own achievements and insight inspire lagging strength. Without such association, the teacher inevitably suffers from too continuous intercourse with his inferiors in experience, and is in the gravest danger of losing sight of the true standards by which to measure his own worth. From living too long with his nose too close to a grindstone, he loses perspective, and failing to see true values, fails to be able to impart them in his teaching. To be the best informed among the ill informed, is often a calamity. To be an appreciative student among students in any department of knowledge, is a great and inspiring privilege.

Every intellectual man has an avocation as well as a vocation, and the real man is more often revealed by his avocation than by his vocation. I would suggest that the teacher of biology find his avocation in study and research in his own field. Then he will grow in knowledge as in his power to impart, and the older teacher will be the better teacher instead of being limited to what he can find in an old text-book, and tolerated only until he can be retired on a pension fund.

Teachers should be students, for the pupils' highest opportunities are determined by the teacher's limitations. Study and research keep the teacher in the attitude of a learner and by so doing, keep him sympathetically in touch with the student attitude of his pupils.

Moreover, Biology is essentially a subject of relations; the knowledge of the inter-relation of the different fields of biological science can be in no way so well appreciated as by striving to know some one subject exhaustively, and the whole subject as growing science.

The presentation of biological data in their relations, is part of the very subject matter of biology, without which it ceases to exist as a science and becomes a mere exercise in observation. A student recently remarked to me, that he had learned more in two weeks from a celebrated surgeon in a hospital in Vienna than in any year of previous study, because his professor taught anatomy in relation to embryology. The details of the subject as intellectual stock-in-trade lose their larger value out of relations, and can be presented in their true relations only by those who know their value in a larger scheme of knowledge.

I believe that every teacher who keeps his enthusiasm alive through the years of professional work, has as the source of his inspiration, an intense belief in the value of the subject matter he teaches; and it seems to me that the teachers of Biology have exceptional opportunities of continued contact with the original sources of inspiration in research; through study which may mean not only their own progress, but which may contribute in some small measure, however remotely, to the establishment of some great truth.

Teachers are the middle-men between closeted scholars and the busy world. The more they can bring of scholarship to their teaching, the more they can contribute towards the creation of an educated public. Knowledge is worthless if it be not used as an instrument to dispel ignorance, and, other things being equal, the best scholar is necessarily the best teacher, because he has more to impart. To one, the most useful form of biological knowledge may mean economic questions of relationship; to another, familiarity with the forms about him; to some, physiology in its more utilitarian phases, or in its larger sense; while others may value some more purely scientific phase of the science, such as evolution through variation, or the laws of heredity.

But whatever line of work we pursue, let us remember that the more helpful methods of teaching are the broader ones—those which open avenues of suggestion to the teacher and pupil alike. If we are teachers of students of more mature capacity, let us strive to be worthy leaders of a good cause. If our pupils are inappreciative through ignorance and immaturity, let us, while we try to illume their darkness, strive to hold to our own high standards for the preservation of our own intellectual lives, and resist with the whole force of body and mind the depressing atmosphere of ignorance.

In conclusion, teachers are unworthy of their high calling who cease to grow in the field of knowledge in which they profess to be masters. They are unworthy as ideals for the young, whose best opportunities they represent, when they cease to be students.

It is to be deplored that the conditions under which secondary teachers have to work, afford so little opportunity for the nurturing of the intellectual life, but it devolves all the more on the individual to wrest from importunate service a tithe of time in which to keep pace with progress. This can only be done by biologists, through continued graduate study and research.

SOLUTION BY FACTORING OF PROBLEMS UNDER THE FACTOR THEOREM.

By Carl Ackermann,

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Texts in elementary algebra published during the past few years have been making use under the name of the Factor Theorem of a principle long used in the solution of higher equations with commensurable roots as presented in college algebras generally. By this method in the latter texts commensurable roots are found either (1) by actual substitution of the supposed root, or (2) by synthetic division, or (3) by method of divisors. The first of these methods seems to have been found the only feasible one when the subject is taken up so early in a course as in the treatment of factoring in elementary algebra. Nor has the writer any complaint to make against this practice. There are indeed some reasons why the factoring of cubics, etc., of the character treated under the factor theorem should be relegated

to a place later in the course, yet the student comes across just such problems often early in his work and does not know how to deal with them. Some of the old algebras gave us a makeshift for their solution, but nothing really satisfactory was offered till the publication of the factor theorem.

So far all is well. But the writer feels that a better factoring process should be substituted for the division that is recommended for the securing of the other factors. He has used the principle of the factor theorem for years, and when beginning to use it sought for a factoring process to complete the work. He has found that students readily grasp it and that his pupils are glad to use it in their teaching.

Below is given a solution which he believes clearly presents the method of solution. The method of finding the first factor is known or can be easily found in almost all late texts and needs no treatment here.

Factor: $x^3-10x-3$. Substituting -3 for x, the quantity vanishes. Therefore $x+3$ is a factor. Bring down the first term of the quantity to be factored, x^3. Then in order to have the factor $x+3$ in the first two terms I must add $3x^2$ as a second term, and in order not to change the value of the whole quantity I must subtract the same quantity. My problem then takes this form:

$$x^3+3x^2-3x^2-10x-3$$

Grouping now we have

$$(x^3+3x^2)-(3x^2+10x+3)$$

We know that $x+3$ is a factor of both groups because it is a factor of the whole quantity. Factoring therefore we have:

$$x^2(x+3)-(x+3)\ (3x+1) \quad \text{or}$$
$$(x+3)\ (x^2-3x-1)$$

This method will factor any cubic in which a factor of the form of $x \pm a$ appears.

AN ATTEMPT TO CORRELATE ALGEBRA AND PHYSICS.

By Willard S. Bass,

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This attempt to correlate some of the work in algebra and physics is an outgrowth of two opinions held by the writer. The first is that the abstractions of mathematics are most easily and naturally acquired by building them up from perceptions of relations of concrete material objects. The second is that, conversely, the relations of matter and energy, which it is the province of physics to study, are best understood and most thoroughly mastered when they are not only seen in special cases, but also perceived as one phase of a more general law. The wider scope of the law as seen by the pupil and the more numerous its applications which he knows, the better has the special case been taught.

Since the function of mathematics is the expression of general relations of quantity, and the deduction of one set of relations from another, an adequate understanding of the principles of physics involves the use of considerable mathematics. As a natural corollary from the two opinions above, it follows that the most economical method of obtaining the mastery, both of the relation of the concrete objects and of the abstraction deducible from it, is by a thorough study of definite cases which exhibit the relation in the concrete and involve the desired abstraction.

The method of instruction which is suggested, is for the pupil to study the concrete case long enough to understand it thoroughly and to find some relations which seem to him both true and important, and then to state these relations in the abstract language of mathematics. The abstract statement should then be expressed in several forms and be used many times in practical applications and concrete problems, until the pupil is thoroughly familiar both with the abstraction and with the mathematical machinery necessary for its use. Thus, while the pupil works upon the mathematical side of his problem, he deepens and makes more exact his knowledge of its physics; and, while he is studying his particular case, he is seeing a physical embodiment of his mathematical concepts, which serves to make the latter more definite, more real, and more significant.

This mutual reinforcement of the two studies of physics and mathematics ought to result in better work in both and in a saving of time to the pupil as well. But a practical difficulty lies in the way of teaching by this plan. A very definite sequence of topics in algebra has been almost universally established, and, in truth, the logical connection necessary to a course in mathematics seems to render any radical departure from this order impossible. Teachers of physics also take pleasure in the unity of their science and like to follow a sequence in its teaching which brings out its logical connections and nice inter-relations. It would be nothing short of a miracle, if these two sequences were such that the two courses could be placed side by side and constantly reinforce each other. The miracle has not happened; on the other hand it will require a long period of adjustment before the courses render each other the assistance which they should. It is with the hope of hastening the time that the writer offers the following as one arrangement of a part of courses designed to increase their mutual helpfulness, while maintaining the necessary sequence of algebra, and doing no great violence to that of physics.

In order to understand the following outline of the work, and to form a just estimate of the bearing of its results upon the opinions expressed at the beginning of this paper, and of its application, if any, elsewhere, a little explanation of the circumstances under which the course was given is necessary.

The entire course, i. e., in both subjects, was in the hands of one teacher. Seven periods per week of an average length of about forty minutes were given to the subject. On days when two periods came, they were made consecutive and were generally used for laboratory work. No specific periods per week were given to mathematics and others to physics, but at each recitation the topic which seemed at that time most necessary to the progress of the class was taken up, regardless of its classification. In the outline, however, the topics are classified under physics and algebra. This is only to show more clearly the outcome. The approximate order of presentation can be seen by reading the outline across.

The usual method of treating a topic was: (1) An experiment with material in which the pupil sought the answer to some particular question given by the teacher. (2) The study and interpretation of the results, graphs being used here when helpful.

(3) The answer to the question proposed in (1) which was stated in the form of an equation when possible. (4) A further study of the mathematical processes involved in (2) and sometimes their extension to similar cases. (5) Problems involving either the mathematics of (2), or the law of (3), or both.

The class at the beginning of the course had had what might be called a half year's work in algebra. They had had the fundamental operations with signed numbers, could manage easy fractions, solve simple equations, and had plotted several linear graphs. It will be at once seen from the outline that the first work of the course was designed to give a review of these topics.

OUTLINE OF COURSE IN PHYSICS AND ALGEBRA.

PHYSICS	ALGEBRA
I. Friction.	$F = k\,W$. Derivation of law from data. Graph. 1
Lever. II. Parallel Forces.	$\Sigma F = O$. Addition of algebraic numbers. Problems.
III. Principles of Archimedes.	
IV. Density of a Heavy Solid.	$D = \frac{W_a}{W_a - W_w}$ Derivation of formula from definition. Computation. 2
V. Density of a Light Solid.	$D = \frac{W_a}{W_a - W_w} = \frac{W_a}{W_a - (W_{B+S} - W_s)_w}$ Algebraic subtraction. 3
VI. Specific Gravity by Flotation.	$S\,G = \frac{\text{Wt. of body}}{\text{Wt. of same vol. of water}} = \frac{\text{Length of submerged part}}{\text{Length of whole}}$
VII. Density of a Liquid.	Problems.
Definition of Turning Tendency. (Moment.)	$T = d \cdot f$. Convention of signs of d, f, and T. Law of signs in multiplication. 4

VIII. Sum of Turning Tendencies Acting on a Bar.	$\Sigma T = O$. Problems in equations with one unknown.
IX. Center of Gravity of Irregular Body. Weight Moment of a Bar.	Center of gravity of a triangle. $T_w = W \cdot d$ 5 Problems. 6
X. Inclined Plane. Definition of Work, Energy, Mechanical Advantage. Screw.	$L \cdot D_L = P \cdot D_P$ $M A = \frac{\text{Load}}{\text{Power}} = \frac{\text{Height}}{\text{Length}}$ or $= \frac{\text{Height}}{\text{Base}}$ $M A = \frac{\text{Circumference}}{\text{Pitch}} = \frac{2 \pi r}{p}$ Determination of π by experiment.
Belts and Pulleys.	Proportion; direct, inverse, squares, cubes, theorems.
	Simultaneous equations. Graphs. Elimination.
XI. Pendulum.	$t = k \sqrt{l}$ Graph. Square root. Cube and higher roots. Logarithms. 7
XII. Breaking Strength of a Wire.	$B S = \frac{\text{Force}}{\text{Area}} = \frac{f}{\pi r^2}$
XIII. Elasticity of Bending.	$B \propto f$ or $B = k f$
XIV. Same. Effect of Varying Dimensions.	$B \propto L^3$, $B \propto \frac{1}{W}$ $B \propto \frac{1}{T^2}$; $B \propto \frac{f L^3}{W T^2}$
XV. Breaking Tendency.	$Br = \Sigma T$ (on one side). For Beam with uniform load $Br = -\frac{1}{2} W X^2 + \frac{1}{2} W X$.
XVI. Surface Tension. 8	Quadratic equations. Solutions by factoring graphs, completing the square.

XVII. Atwood's Machine. Time and Distance.	$S = \frac{1}{2} a t^2$. Graph.
XVIII. Atwood's Machine. Time and Velocity.	$V = a t$. Graph. Computation of g.
Laws for a Free Falling Body Having Initial Velocity U.	$S = u t + \frac{1}{2} g t^2$. $V = u + g t$. Problems. 9

NOTES UPON OUTLINE.

1. The friction is equal to the weight times a constant. Graph is used to verify the law by a visual comparison with the data.
2. Density is equal to weight of body in air divided by the difference between its weight in air and its weight in water.
3. The body being lighter than water, is weighed in water with the help of a sinker upon which it exerts an upward pull. The weight of the block and sinker is less than that of the sinker alone and therefore $W_{B+S} - W_S$ is a negative quantity.
4. Here T = turning tendency, F = force and at the distance of the axis of the bar from the line of the force. The experiment furnishes a clear proof of the law of signs in multiplication for the particular quantities it considers.
5. Turning tendency due to the weight of a bar equals the weight multiplied by distance of center of gravity from axis of rotation.
6. These problems involved a review of all that had preceded and the solution of many simple equations.
7. Logarithms are useful at this point in the course and frequently later. The pupils can understand the method of their use, but have difficulty in acquiring sufficient facility to make their use a saving of time.
8. This experiment serves to compare the properties of liquids with those of solids. It is not connected with the mathematics.
9. These problems include not only problems of falling bodies but many others involving quadratics.

In the outline the numbered articles are subjects of experiments performed by the pupils individually. Of these eighteen experiments, twelve are on the usual lists of high school experiments, and the other six are of equal, or sometimes greater difficulty. The class worked a large number of problems, but did very little text-book work—less than I would have another class do in similar circumstances. If an estimate were asked of the amount of work done by the class in physics, I would say about two-fifths of that ordinarily accomplished in a year by a high school class. The work in algebra varied somewhat, both in matter and treatment, from that ordinarily given, most noticeably in the attention given to ratio, proportion, and variation, the inclusion of logarthims, and the neglect of radicals. These variations

make an estimate of the quantity of work done rather difficult, but in the writer's opinion it was a full year's work. These two estimates show a total of one and two fifths years' work done in seven periods a week for a year. This shows a normal return for the pupils' time—no gain and no loss. As this is the first time that this particular road has been tried, it would, therefore, seem likely that in a second journey some of the obstacles met in the first could be avoided and some short cuts found. If this should be the case, there would then be some actual saving of time for the pupil.

In estimating the value of a method of teaching the quality of results is of more importance than the quantity. But differences in quality of mental results are harder to estimate than those of quantity, especially as different classes present such different natural capacities. After making as careful allowance as possible for this personal equation, of the observed as well as of the observer, it appears that the class gained the following advantages from the teaching of the physics and algebra together: (1) Clear mathematical concepts; (2) power to see the mathematical relations of concrete objects and actual problems; (3) knowledge of the usefulness of algebra. The frequently heard question "What is algebra good for anyway?" did not come up. The presence of problems to be solved answered it before it was asked.

Against these gains one loss may be placed. Some of the pupils at times felt at sea as to what to call algebra, and what to call physics, and were more or less disturbed by it. Now, the two sciences are in many cases but two aspects of the same truth, and whether one shall call a given problem physics or algebra frequently depends only upon whether one is more interested in the subject matter of the problem, or in the means by which he reaches the answer. So, in fact, the pupil's vagueness about a definite demarcation between the two subjects is less vital than his mental unrest about it. The latter, so far as it exists, is a distinct detriment to the progress of the pupil. It was, however, very largely dispelled by explanations like the above, and with more careful presentation of the subject matter. I do not think that it need arise at all.

A GUIDE TO THE EQUITABLE GRADING OF STUDENTS.

By Winfield Scott Hall, Ph. D., M. D.
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This study was undertaken in response to the following inquiry of a prominent teacher in one of our secondary schools:

"Does science offer any solution of the problem, which confronts every teacher from the kindergarten to the university, of the equitable rating of pupils?"

In the first place the question might be raised as to the advisability of rating pupils at all.

Most educators would agree that the grading of kindergarten pupils on a percentage basis would be folly. The rating of pupils in every school exercise in the grammar or high school and the marking of college and university students to the fraction of a percent brings marking into greater prominence than it deserves and incidentally makes a slave of the teacher. It gives the pupil false values, leading him to look upon his mark as the goal for which he is striving; while in reality it serves only to indicate the teacher's estimate of his progress toward the goal.

On the other hand it seems necessary to make periodic estimates of the pupil's progress in the lower schools and to report these to the parent, to the end that in case the pupil is doing unsatisfactory work the parent may coöperate with the teacher in seeking the cause and removing it, thus bringing the pupil up to the standard. In any case a judicious rating of the pupil serves as a wholesome stimulus.

Official records of the character of work done are valuable in the transfer of pupils from one school to another, or of students from one institution to another.

The standing of students in colleges, universities, technical or professional schools as indicated by the records serves in a large proportion of cases as a basis for their appointment to positions; and because of this becomes a very strong stimulus to students in such institutions.

Granted then, that some system of marking is a practical necessity, let us choose such a system as shall require the least possible time on the part of the teacher and at the same time serve as an

equitable estimate of the pupil's development. This brings us to the question quoted at the beginning of the paper.

Science offers the following solution:

I. To mark an examination paper or any school exercise is to measure psychic function.

II. The measurement of functions as well as of structures of the human subject yields "anthropometric data."

III. All anthropometric data obey *the law of distribution of biologic data.*

These propositions, if not axiomatic, are so nearly self-evident that they need no demonstration. They may, however, be illustrated.

What is meant by the law of distribution of biologic data?

Let us suppose that a school medical examiner has recorded upon cards the height, weight, head girth, chest girth, lung capacity, acuteness of vision and of hearing, for 5,000 school children. Each individual is represented by a card. The individuals may be grouped first as to sex. Let each sex be grouped as to age, and each age as to nationality, or social position of parents. Suppose there are 314 boys of 16 years representing families which have been for several generations in America. Such a group of individuals could be looked upon as *a homogeneous group.* Let us arrange the cards in groups, putting in one pile all whose head-girth is 50 cm. or fraction of a centimeter; in the next group, cards which bear a record of 51 cm. and so on, the result being shown in the following table:

TABLE I. GIRTH OF HEAD OF 16-YEAR-OLD AMERICAN BOYS.

Measurement	50+	51+	52+	53+	54+	55+	56+	57+	58+	59+	60+	61+
No. of observations	1	3	13	28	34	73	69	52	26	11	3	1

As one studies this table it becomes apparent that in the matter of head-girth Nature seems to be aiming at a type (shooting at a mark). Some of the attempts are wide of the mark while most of them are clustered close around the mark, thinning out with increasing distances. If a marksman were to aim 314 shots at a dot in the center of a target he would find that these shots would be grouped about the center in a way analogous to the grouping of the above tabulated measurements of head girth around the median value.

Let the bull's-eye of the target be obliterated and the mathematician would have no difficulty in determining what its location had been, simply through a study of the grouping of shots about it. In a similar manner the anthropologist has no difficulty in locating the mark (Median Value) at which Nature is aiming in the head-girths of such a group as the above.*

But this question of the median value is broader than a simple statistical problem—it is based upon fundamental, biologic and mathematical laws. One could hardly follow the steps in the above presentation without noting that the number of observations for each separate value increases progressively from the two extremities toward the middle of the series where the maximum is represented by one large value or by two large and nearly equal values. Francis Galton called especial attention to this and published figures to illustrate this law of distribution of values. The law may be formulated thus: Any structural dimension or functional property of any species of living thing tends to approximate a fixed standard or middle value, while the other values will progressively shade down to a maximum in one direction and to a minimum in the other. This may be called the *law of distribution of biologic data,* understanding, of course, that it is the distribution of data concerning any one structural feature or functional property within any one species of plants or animals.

Table II shows the distribution of head-girths of 1,071 youths from 16 to 19 years old.

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* The *Median Value* is the value represented by the median measurement of a series of measurements; that is, that measurement which has as many values above it as below it.

The total number of observations being 314, we have only to locate and determine the value of the 157th measurement from either end. Adding the observations from left to right (1+3+13+28+34+73=152) we find that the middle individual must be in the group whose head-girth is 56+ cm. This may be called the *Median Group*. Moreover, the middle individual is the fifth one in this group from the left, or the 64th from the right. Now the 69 individuals of this median group have head-girths ranging from 56 cm. to 57 cm. and according to fundamental biologic laws, they will be practically evenly distributed through this one centimeter. The fifth individual from the left must then have a head-girth of 56 5/69 cm. or 56.072.

If it is desired to reduce this simple process to a mathematical formula, that can be readily done:

Let n = the total number of observations (314); m = the number of observations in the median group (69); l = the sum of the observations at the left of the median group; a = minimum value of the median group (56 cm.); d = arithmetric difference in the minimum values of the groups (d = 1 cm. in above) and M = median value to be determined. Then:

$$M = a + \frac{d\left(\frac{n}{2} - l\right)}{m}$$

$$M = 56 \frac{\frac{314}{2} - 152}{69} = 56 \frac{5}{69} = 56.072$$

TABLE II. HEAD-GIRTHS, BOYS 16-19.

Measurement	51 cm	52	53	54	55	m 56 56.46	57	58	59	60	61	
No. of observations	4	23	59	108	224	257	230	110	38	16	2	=1071
Coeff. $(a+b)^{10}$	1	10	45	120	210	252	210	120	45	10	1	=1024

The distribution of these measurements illustrates the above formulated law. The computed median value is 56.46 cm. Note that of the 1,071 observations, 711 of them fall within 1.46 cm. below and 1.54 cm. above the median value, while the 360 remaining observations progressively shade down to a maximum (61 cm.) in one direction (110-38-16-2) and to a minimum (51 cm.) in the other. Figure 1 shows the 1,071 cards distributed in the different groups as indicated in Table II.

Table II shows a third series of values, viz., the coefficients of a binomial raised to the tenth power. If the expression a + b

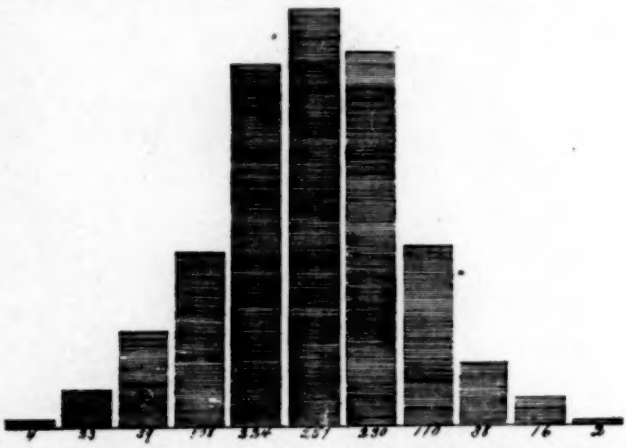

Figure 1.—Showing the 1071 cards grouped as to the size of head-girth; the middle stack shows that 257 boys had a head-girth equal to 56 cm., but less than 57 cm. See Table 2 for other facts regarding figure.

were squared—$(a^2 + 2ab + b^2)$—and this multiplied by (a + b) and so on to the tenth time, an expression of eleven terms would result and the coefficients would be 1-10-45-120, etc., as shown in the table. The sum of these coefficients equals 1,024. If one compares this series of coefficients with the series of observations in the line above them he notes a remarkable parallelism. Now this is not the result of a rare chance; it is in conformity to a natural law. *It transpires then that the law of the distribution of*

biologic data is the law of the series of binomial coefficients The correspondence of these series is made graphic in Figure 2 where the values are plotted in the form of a curve. Even with a thousand observations, the curve of distribution corresponds very nearly to the binomial curve: if there were ten thousand measurements it is certain that the two curves would be perfectly coincident.

The odd powers of a binomial 1st, 3d, 5th, 7th, etc., have an even number of terms and two equal maximum coefficients instead of one maximum coefficient. This feature of the binomial series also corresponds to something very frequently found, namely, that there is no one pile that is clearly the maximum pile, but that there are in the middle of the series, two maximum piles of fairly equal size. For an example of this, one only has to refer to Table I where there are 12 terms, the two middle terms being nearly equal (73-69) the values decreasing rapidly in either direction from these median groups.

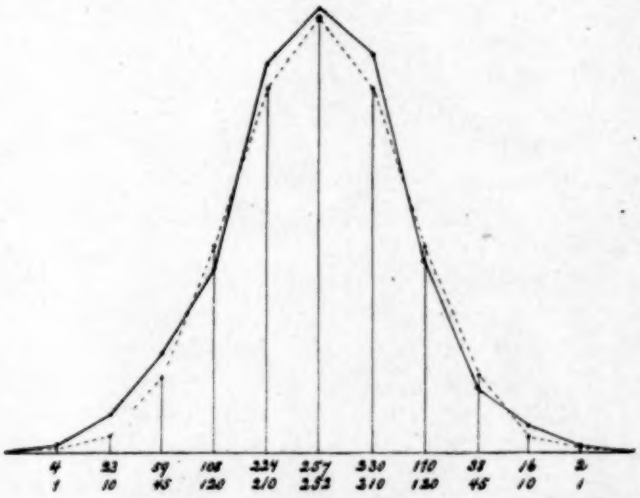

Figure 2.—Showing the relation between the curve of biologic distribution and that of the binomial coefficients. The former is represented by the continuous line plotted from the upper line of the figures and represents the distribution of 1071 boys with reference to girth of head: the median girth being 56.46 cm. The curve of binomial coefficients is represented by the dotted line and is plotted from the coefficients of $(a+b)^{10}$. This set of coefficients being chosen because their sum (1024) most closely approximated the number of individuals represented in the biologic distribution curve.

What is the significance of this remarkable coincidence between the curve of binomial coefficients and that of the distribution of biologic data? This is a problem the solution of which belongs to the biologist rather than to the mathematician. By

way of suggestion, simply, the reader may be reminded: (1) that the principal biologic factor in determining specific characters is heredity; (2) that heredity involves a double influence—the father plus mother—which may be represented mathematically as (a+b); (3) that when a value is influenced by a series of variable factors it is found in mathematics that it would vary as the product of these factors. The only way heredity can act in a series of influences is in the series of succeeding generations which may be represented as $(a+b)\times(a+b)$ etc., viz., the paternal, (a), and the maternal, (b), influences of the successive generations. The total influence of heredity could then be represented by $(a+b)\times(a+b)\times(a+b)$ to the nth power. If the coefficients of this product be plotted they will describe the curve shown in Figure 2. If the numerical data arising from the measurement of any one feature of the lineal descendants of the nth generation of an original father and mother (a+b) be grouped as in Table II and the numbers plotted as in Figure 2, the curve will closely approximate or actually coincide with the binomial curve.

The Belgian astronomer and anthropologist Quételet[1] was first to define and determine the *Median Value,* and to emphasize its advantages over the arithmetical mean or average. Francis Galton[2] the English anthropologist elaborated the idea and applied it in his researches as did also Stieda[3] the German anthropologist.

In his studies on the "Changes in the Proportions of the Human Body during the Period of Growth," the writer[4] applied the method of Quételet and further elaborated it. Later the writer[5] added still further to the technique, and demonstrated the coincidence of the curve of distribution of biologic data with the curve of binomial coefficients, as set forth above.

The question naturally arises: what has all this to do with the equitable rating of students? It has everything to do with

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1 *Quetelet:* "Anthropometrie. on mesure des diff. facultes de l'homme." Bruxelles, 1870.

2 *Galton:* "Natural Inheritance," 1889, London.

3 *Stieda:* "Ueber die Anwendung der Wahrscheinlichkeitsrechnung in der anthropologischen Statistik," Arch. f. Anthrop., 1883.

4 *Hall:* "Changes in the Proportion of the Human Body," Jour. Anthropological Institute, London. 1895.

5 *Hall:* "The Evaluation of Anthropometric Data," Jour. Am. Med. Assoc., December 21, 1901, Chicago.

it. All numerical data from the observation of either functional or structural characters or features of the human subject obey the law of distribution of biologic data. The rating of students is a measurement of psychic function and yields numerical data. These data must therefore obey the law of distribution of biologic data.

In order to put this proposition to a test, the writer procured from the registrar of the institution with which he is connected all the records turned in from the writer's department for the past decade. These records represent the semester rating of the students in the department of physiology. The rating shows the estimate of the student's work as represented by his laboratory work, note-books, laboratory conferences, oral and written quizzes and final examination, only one mark being reported each semester for each student. This one mark might represent the summation of as many as thirty or forty ratings or as few as eight or ten.

The accompanying table (Table III) shows the classified results of each semester's report—for example, on May 27, 1896, I reported on 51 students, giving one of them AA (95-99); fourteen, A (90-95); fifteen, BB; seven, B; seven, CC; six, C; and one E.

The total number of students reported during the decade was 2334. Note that the total number receiving E was 40; D, 122; C, 223; CC, 287; B, 477; BB, 594; A, 468; AA, 116; AAA, 7.

Comparing this series of totals with the series in tables 1 and 2, one will be struck with the fact that like those series the numbers increase from the minimum rating (E) to the median rating (BB) and then decrease to the maximum rating (AAA). Of the 2334 students, 1539 or about two-thirds of them were rated between 80 per cent and 95 per cent, a range of 15 per cent in rating, while the remaining one-third were distributed over the two extremes and covered at least a 30-per cent range of rating.

The series: 40-122-223, etc., may be compared with a series of binomial coefficients of the same number of terms and graphically represented by curves. There are nine terms in the series; $(a+b)$ to the eighth power will yield a series of nine terms: 1-8-28-56-70-56-28-8-1.—the sum of the series being approximately one-tenth the sum of the above series of student-ratings

TABLE III.

Table 3.

	EXAMINATION	E Below 60	D 60-70	C 70-75	CC 75-80	B 80-85	BB 85-90	A 90-95	AA 95-99	AAA 99-100	TOTALS
I	May 27, 1896	1	0	6	7	7	15	14	1	0	51
II	June 11, 1897	1	3	11	11	13	19	12	0	0	70
III	June 14, 1897	0	0	9	9	22	23	19	2	0	84
IV	June 1, 1898	2	0	9	9	21	21	5	0	0	67
V	June 14, 1898	1	5	19	19	18	18	7	7	0	88
VI	June 8, 1899	1	5	2	3	1	26	10	0	0	48
VII	June 19, 1899	0	1	9	10	21	23	11	0	0	75
VIII	1st Year Feb. 2, 1900	0	5	4	7	8	18	18	2	0	62
II	2nd Yr. March 10, 1900	0	0	1	3	9	10	6	9	0	40
X	1st Yr. May 31, 1900	0	6	4	5	14	13	21	3	1	72
XI	1st Yr. Feb 6, 1901.	0	2	8	6	13	18	12	5	0	64
XII	2nd Yr. March 9, 1901.	4	2	4	5	8	12	16	10	1	62
XIII	1st Yr July 1901.	2	2	7	7	12	17	13	10	3	73
XIV	2nd Yr. March 6, 1901.	0	0	3	13	20	22	18	5	0	81
XV	1st Yr. Feb. 28, 1902	0	6	15	9	26	19	23	2	0	102
XVI	2nd Yr. Feb 28, 1902.	3	9	14	8	22	19	24	8	0	102
XVII	1st Yr. June 4, 1902.	7	10	13	22	23	22	20	6	0	123
XVIII	1st Yr. Feb. 7, 1903.	8	12	10	13	26	30	26	2	1	128
XIX	2nd Yr. Feb. 18, 1903	1	10	9	16	28	22	39	5	0	130
XX	June 8, 1903	4	6	12	21	35	52	15	5	0	150
XXI	2nd Yr. Feb. 15, 1904.	4	8	14	20	29	26	26	5	0	132
XXII	1st Yr. Feb. 16, 1904.	0	3	12	22	19	25	17	8	0	106
XXIII	June 1, 1904.	0	9	15	5	16	27	23	10	0	105
XXIV	Jan. 30, 1906.	0	9	9	11	22	33	23	4	0	116
XXV	Feb. 3, 1905.	0	5	11	7	20	31	18	4	0	96
XXVI	1st Yr. June 16, 1905	1	4	9	14	22	23	20	3	1	107
		40	132	233	293	477	594	468	116	7	2334

they have only to be reduced to common terms and plotted. (See Fig. 3.)

That the curve derived from the rating of the 2334 students is really a binomial curve no fair-minded judge would for a moment question or doubt. We have, therefore, demonstrated beyond cavil that examination data is biologic data and obeys the laws of distribution of biologic data.

Certain important divergences from strict coincidence remain yet to be explained. Why does the apex of the curve stand to the right of the symmetrical binomial curve; i. e., why is the curve of my ratings unsymmetrical? The answer is to be sought in two directions:

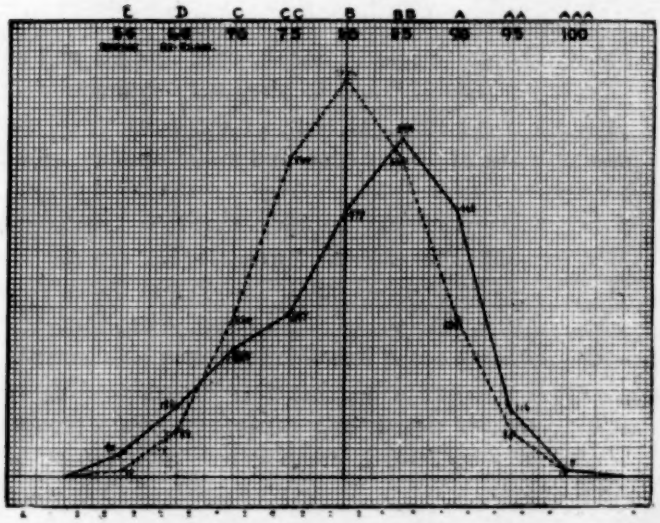


Figure 3. Showing relation between the binomial curve (dotted) and the curve derived from the rating of 2334 students.

1. Either the examiner was too generous and habitually rated his students above their equitable deserts; or

2. The students were (in a sufficient number of individual cases to influence the totals) guilty of raising their rating above what it should be by nature through dishonest means or extraneous aids in quizzes, examinations and the preparation of notebooks.

I am convinced that both of these factors were at work, and in the same direction, i. e., both tended to raise the rating of students and thus to throw the curve out of symmetry. Together these two factors have made a difference of approximately 5 per cent—the actual median value of the rating of the 2334 men being 85.15 per cent when theoretically it should have been 80 per cent.

The problem which we are attempting to solve is to discover a guide to the equitable marking of students. The solution is at hand. Suppose one has a class of 128 students, the above offers him a guide as to how their marks should be distributed. The sum of the coefficients of a binomial $(a + b)$ to the seventh power equals 128, and the series is as follows:

Coeff. $(a+b)^7$	1	7	21	35	35	21	7	1	Computed number of individuals to receive the marks indicated below.
Rating	E	D	C	CC	B	BB	A	AA	

One would expect, if the class in question is an average class, and if they would do absolutely honest work in class and examination, and if the teacher could be absolutely equitable and possessed of good judgment, that eight of the class would fall below a passing mark of 70 per cent, and a like number would rise to 90 per cent or above.

Classes vary as do individuals—though through a much narrower range—for example the classes represented in ratings X-XIII (Table III) were two consecutive classes that from their entrance upon work in the institution to their victorious competitive hospital and state board examinations showed unusually high records. On the other hand the class represented in ratings XX and XXI had the reverse experience. One must then be prepared to find a moderate divergence from the typical in any class. In a decade, however, these divergences will balance each other so that any lack of symmetry in the curve of ratings may be safely ascribed to the other variables—the lack of perfect honesty on the part of the pupil and the lack of perfect judgment on the part of the examiner.

The writer has been asked how he would account for the failure of 30 per cent of a class in a semester's work. The answer is: There must be something wrong with the class or with the teacher—probably the latter as divergences from the normal are far more likely to be noted in individuals than in classes.

SUMMARY.

1. The marking of students is a measurement of psychic function, and yields biologic data.

2. All biologic data—including student ratings—are distributed about a median value in conformity with the *law of distribution of biologic data,* which law is demonstrated to be represented in *the binomial curve.*

3. Average classes of students, doing honest work and marked equitably will yield results which when tabulated should conform to the binomial curve, i. e., the number receiving medium marks should far exceed the number receiving high or low marks.

GEOMETRY IN THE CLASS ROOM: PURPOSIVE GEOMETRY.

By Arthur Latham Baker, Ph.D.,

Manual Training High School, Brooklyn, N. Y.

The four great pedagogic factors in the study of any science are:

What specific thing the student is trying to do.

Wherewith he is doing the work.

Why he selects certain processes.

How he builds up his subject.

Our text-books in geometry answer the *how* very thoroughly and completely, but they neglect almost entirely the other three factors. This is a serious pedagogic error. From the *how* alone few students get any real idea of the other factors, and that little with great difficulty and lack of system. The *how* alone is mere parrot teaching and parrot learning.

On the other hand, the student who knows the *what, wherewith,* and *why* can easily build up the *how* for himself.

There are two fundamentally different methods of presenting the facts in geometry, the dogmatic and the heuristic; the *how* and the *why* methods. The former takes you by the nose and leads you in one door and out the other and justifies the process by the logical accuracy of the result. The latter says we will not pass that door until we know the reason for so doing, the exact purpose of the step.

Until comparatively recently the former method even carried the student; Nothing was left for the student but to sit in acquiesceful appreciation while the logical accuracy of the process was passed before his eyes. This has been improved by changing the carrying into leading by suggestive questions, directive schemes, etc., so that now the student has to do his own walking; but he is still led, and the process justified by the logical safety of his arrival.

The suggestive questions were a great advance but there still remained the puzzling question, *what suggested the suggestive questions?* Until the impulse which suggested them is known the student is still in leading strings, the blind follower of a blind leader.

We should go a step further, and let the student refuse to take a single step, pass a single door, until he has a clear idea of the purpose and the possible results of the step. Then the student will no longer need our guiding strings. Knowing the purpose and the possible results he will be able to choose for himself which door to pass through to attain a required result.

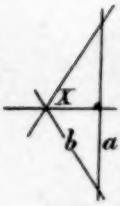
I do not know that I can better contrast the relative pedagogic importance of the two methods than is the words which I take from the Pedagogy of Herbart, Ufer (Heath & Co.) p. 82: "The pupil must know from the beginning *what is aimed at* if he is to employ his whole strength in the effort of learning; and he will employ it provided he knows definitely what is to be reached. To lead him up unconsciously by question and task whose *purpose* he does not clearly see, has the disadvantage that neither a free rising mental activity nor a clear internally connected insight takes place."

In plain English *no student should be asked to do anything which he does not clearly see the purpose of. Every demonstration in the geometries is a violation of this pedagogic law.*

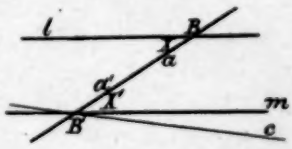
In the familiar early proposition, only one perpendicular to a straight line from a point without, the first step in the proof is, fold the diagram, etc. Why? For what purpose? Not the slightest suggestion is given in any book. So in the proposition, alternate interior angles of parallels, etc., through the middle point of the transversal draw a perpendicular to the parallels. Why, for what purpose? The books fail to give any purpose. Not only that, but the construction is absolutely purposeless at this stage of the demonstration. It is not until we are half way through the demonstration that it is needed or can be used. In finding the length of the bisector of the angle of a triangle, why draw the circumscribing circle, as directed at the outset? It is absolutely purposeless and useless until we reach a point initially clear around the corner and out of sight, and unthought of.

And so on, and so on, demonstration after demonstration throughout the book.

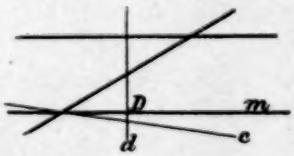
I lay down the proposition, backed by common sense and the teachings of all the great pedagogists, that *every step or construction in geometry should be preceded by a definite purpose* and the student should recognize that purpose; both the immediate and ultimate purpose.

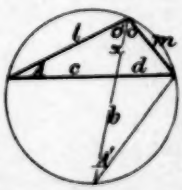


Thus in the proposition, only one perpendicular, etc., we draw the line *a in order* to get a straight line *in order* to show that the other line is not a straight line. The line *b* is drawn *in order* (if possible) to get a broken line, *in order* to utilize the theorem, sum of the angles on a broken line is not 180°. We utilize this theorem in order to say that *X*, the half of the angle on the broken line is not a right angle and because at this stage of the development of the subject, this is the only tool (axiom or proposition) which says anything about angles not being equal. And we make the diagram symmetrical (by folding) *in order* to be able to say that *X* is ½ the larger angle.



So in the proposition, alternate interior angles of parallels, we bisect the transversal for the definite *purpose* of getting a mid point between the vertices so that when revolved, the upper part around this mid point onto the lower, the vertices of the angles in question will come together, so that we can attempt their superposition to test equality. After making this revolution, we find that while *a* and *a'* coincide, and *b* and *b'*, the line *l* falls in some unknown position *c*, though probably as common sense would indicate, on *m*. If this be so, the angle *mc* must be zero. To test this we have only the one tool, two perpendiculars from the same point coincide; and now we make the construction; draw *d* making *D* a right angle for the *purpose* of making *m* one of the perpendiculars, *hoping c* will turn out to also be perpendicular, and therefore the angle zero, which hope the consequences justify.





Again in the problem, to find the length of the bisector of the angle of a triangle. Here the real operation called for is the measurement of a line, which must be done by similar triangles or the Pythagorean Proposition (or their derivatives). The equal engles *O O'* rather suggest similar triangles, though it can be done in either way. Therefore *in order* to get similar triangles we make $A' = A$, whence

$$\frac{l}{x} = \frac{x + h}{m}$$

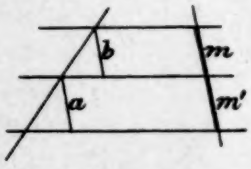
and

$$lm = x^2 + bx$$

But this has introduced a new and unknown quantity *b*. To get rid of this we utilize our list of *suggestive combinations** where the product *bx suggests* the chord of a circle. Now we circumscribe the circle *for the purpose* of replacing *bx* by *cd*, if perchance the vertex of *A'* lies on the circle. The perchance turns out successful, and *bx* is replaced by *cd*, which is in turn eliminated by means of the properties of the bisector of the angle of a triangle.

Had we drawn a perpendicular *in order* to utilize the Pythagorean Proposition, the other tool for measuring lines, we would have won out with equal facility and rather more directly.

Again in the proposition, parallels cutting one transversal into equal parts cut all transversals into equal parts, the operation called for by the conclusion is the comparison of two lines for equality, which may be done by the tool, properties of equal triangles. Hence we make the construction, draw *a, b in order* to get triangles, parallel *in order* to get their angles equal as the first requisite for equal triangles, parallel to *m, m' in order* to get the sides equal as a second requisite. Whence, etc.



These illustrations show what I mean by the immediate and ultimate purpose of the construction. The student makes the construction for a definite immediate purpose, with the contingent ultimate purpose which is just as definite as to purpose though not so certain as to its logical applicability and contingency.

I will now show how the student forms his purposes. He starts with a bundle of indefinite straight lines and a set of geometric axioms (including the general axioms):

(a) Only one straight line between two points.
(b) Straight line is shorter than any other.
(c) Coincident things are equal.
(d) Only one parallel through a point.

Begin simply, with one thing at a time, and build up gradually. He places one line on the plane, and finds nothing metrical (measurable) for discussion. A second line introduces the angle and its discussion, and furnishes, as soon as the straight angle has been identified, the theorem:

*See the author's "Art of Geometry," Sibley & Co., Boston, where the art of building demonstrations is fully set forth.

(e) Sum of the angles on a straight line equals 180°.

(f) Sum of the angles on a broken line is not equal to 180°.

The looks of the diagram very quickly suggest the theorem

(1) Vertical angles are equal,

easily proved by (c) or (e) the only tools which say anything about things being equal.

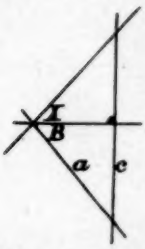
The aspect of the plane, if he is held exhaustively to the possibilities, suggests axial symmetry, and the theorem,

(2) Symmetrical parts are equal,

proved by the identification of axial symmetry as the property of fitting together in pairs when folded along the axis. (see (c).)

A third line in the plane produces something which the two lines could not, line segments and surface segment. By giving various significant and special values to the metrical parts, the line segments and the angles, the student gets corresponding suggestions as to the possible resulting theorems.

One right angle pretty forcibly suggests that the angles of the other sets are not right angles, and that the perpendicular is the shortest line. Suppose he tries the first, the other angles are not right angles, *i. e.*, $X \pm$ rt.$\angle$. The operation called for in the conclusion is $X +$ rt.$\angle$, and the only tool so far which says anything about angles *not* being equal is (f). Therefore, he draws *a, in order to get a broken line,* so that he can utilize (f). He then swings *a* around until $B=X$ (that is, he supposes it done), *in order* to speak of X as $\frac{1}{2}(X+B)$ and utilize the axiom, if the wholes are unequal, the halves are unequal in the same sense. He then draws *c,* prolongation of the perpendicular, *in order* to get a known straight line, so that he can prove that *a* forms a broken line. And the consequences lead to the conclusion, and he has added a new tool to his list, viz.:

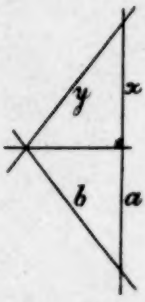


(3) Only one separate perpendicular can be drawn from a point, etc.
Perpendiculars from a point to a line coincide.
Perpendiculars from a point to a line include a zero angle.

Suppose he tries the possible theorem, the perpendicular is less than any oblique line, as seems to be the case from the looks of the diagram.* Here he has to compare two lines

*If he speaks of line segments at all, it must be concerning their length: either $x = y$ or $x \pm y$.

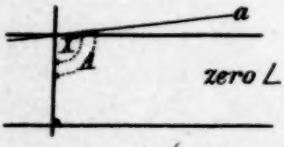
for inequality, $x < y$, and the only tool which says anything about lines not being equal is (b). Therefore he draws a (prolongation of x), and b, *in order* to get two sets of lines connecting the same pair of points, and makes a the prolongation of x *so that* one of them shall be straight, in order to be able to utilize (b). Again he makes the lower half of the diagram symmetrical with the upper half (by folding, etc.), *in order* to be able to speak of x as $\frac{1}{2}(x + b)$ and y as one-half of $(a + x)$; and the consequences produce the conclusion he expected. This adds another tool.



(4) The perpendicular is less than any oblique line.

Suppose he tries 2 rt. ∟s whence he gets the very suggestive and easily substantiated diagram and theorem, proved by (3) or reductio ad absurdum,

(5) Perpendiculars to the same line are parallel.



A right angle and a zero angle suggest the possibility that X is a right angle. But now he finds nothing in his list of tools to measure right angles unless perhaps it be (c), in which case he must draw a making A a right angle *in order* to get a pattern right angle on which to fit x, and thus utilize (c). The consequences of the construction produce,

(6) A line perpendicular to one of two parallels is perpendicular to the other.

One zero angle produces,

(7) Alternate interior angles of parallels are equal,

already discussed.

And so on, by simply specializing the *metrical elements* of the diagram, 3-lines, pairs of 3-lines, 4-lines, etc., the student can evolve many suggestive diagrams with their corresponding results for him to test; specializing with 0°, 90°, = ∟s, singly and in combination, =, ± sides, etc., as illustrated above.

But suppose he had started in with one zero angle, a most seductive entrance, he soon discovers that he needs the measure of a zero angle, and must lay aside the discussion until he finds such a tool. Equal sides, or equal angles, another most seductive entrance, yield (superposition by folding) easy results.

His course of procedure for any proposition should be,

Construction necessary to apply this tool. The selection of the operation and of the tool will furnish the *purpose* of the construction.

Consequences of the construction, or even of the original configuration before any construction lines are drawn. *These should be sought in threefold form.*

1° What geometric figure presents itself?

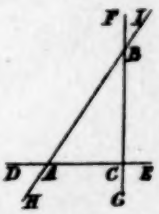
2° What theorems apply to such a figure?

3° What follows from the application of the theorems to the diagram in hand?

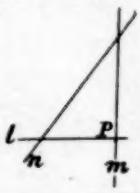
These consequences are suggestively remembered as the 3 F's, *figure, facts, followings.* The question, What propositions apply? will generally narrow the search to a choice of two or three, and *often to a single one.*

What distinctive operation does the conclusion call for; comparison of angles, of lines, of ratios, of surfaces; and for equality, inequality, or measurement?

With what tool is this operation to be performed? For this purpose the tools (theorems) should be closely classified as indicated above.*



Incidentally the student's progress will be facilitated by a legible and sensible notation. In attempting to designate the fundamentals of the hypothesis, we have under the current notation something like this; the indefinite lines HI, DE, FG, and the right angle ACB. It will not do to let the student say the indefinite line AB, for

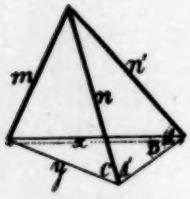


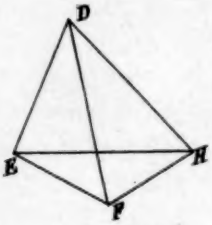
though he will say and write and indicate the "indefinite line AB," he will *think the definite line segment Ab.* The only safety consists in using all the letters shown.

How much simpler to say, the indefinite lines *l, m, n,* and the rt. ∟ *P.* He sees more clearly, thinks more clearly, and, more important than all, he *sees as he thinks and thinks as he sees.*

Contrast the following proofs:

*See the author's "Art of Geometry."

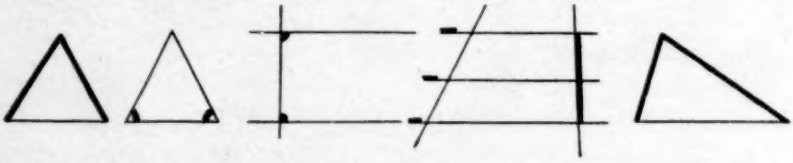


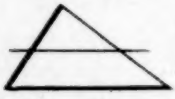
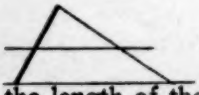


Since DH = DF (Hyp.)	Since n = n′ (Hyp.)
∴ DHF = ∠DFH	∴ A = A′
But ∠DHF > ∠EHF	But A > B
∴ Also ∠DFH > ∠EHF	∴ Also A′ > B
But ∠EFH > ∠DFH	But A′ + C > A′
Much more ∴ ∠EFH > ∠EHF	Much more ∴ A′ + C > B
∴ EH > EF	∴ x > y

One is a torture to the eye, the other can be read almost at a glance.

The foundation of the *what*, the metrical part of the hypothesis, that which is responsible for the particular conclusion in question, should be made patent to the eye by making it heavy, thus:



This shows immediately, at a glance, whether the student is thinking what he draws and drawing what he thinks. It brings to the eye pictorially and exactly what the verbal equivalent is apt to leave hazy and indistinct. It easily detects false concepts which otherwise might be overlooked. Thus in the theorem, A line parallel to the base of a triangle and bisecting one side, if the student presents the diagram  instead of  , it is quite evident that he erroneously considers the length of the base as a fundamental cause of his conclusion. And likewise in the theorem, A line perpendicular to a radius at its extremity, the diagram shows imme-

diately the erroneous idea that the length of the radius affects the question of tangency.

Students are so prone to repeat a phrase parrot like and have some entirely different concept in the mind that any scheme for getting at their mental workings to test whether the mind, lips, and hand are working together or not is a valuable adjunct to the teacher in the classroom.

The division of the letters into *middle letters* for the hypothesis, *end letters* for the conclusion, and *initial letters* exactly in the order of use for the acts of the student in the proof enable the teacher to reach down into the mental arcana of the student and see whether his mind and his outer activities are really working together. A student who introduces the letters *a*, *b* out of order, or who introduces middle or final letters into his proof is evidently not keeping abreast of the argument. For if we make the rule that the statements of the hypothesis and conclusion shall be made in middle letters and end letters only, and that the student shall never introduce any such letters into his proof, we get an insight into the mental workings of the student and are able to test quickly and surely whether his thoughts and fingers are marching together. Thus in the demonstration, alternate interior angles of parallels, etc., if the student puts an *a* in place of the *d*, or in any way alters the order, we see conclusively that he is not using thought but is working from memory only.

This specialized notation gives us three points of contact with the mental arcana of the student, each one telling its own story from its own point of view; the diagram, the lettering, the wording. The three together give a much more perfect picture of his mental concept than any one alone can possibly do, and afford that much greater chance for the detection of false ideas.

So far we have spoken of the activities of the student, the things he does to produce his proof. We ought to probe his premises and find *what are the foundations of his doings;* how many and what premises; and see if he understands the exact foundation of his argument, the causes of the effect. The primary foundation is lines (indefinite), so naturally if we are to reach down to the fundamentals of the subject, we must consider each line as a premise, condition, cause from which the conclusion or effect is to flow. Coupled with these causes are

the secondary elements which flow from the conjunction of the others, line segments and angles. Later when the pedagogic foundations have been started, it is convenient to consider a bunch of lines as one cause. In addition to his bunch of lines he has certain special values of the measurable elements, angles, and sides; these constitute the metrical portion of his hypothesis. For a complete understanding of his premises, he should know how many and what metrical premises are necessary for any particular conclusion. Thus in theorems 3 and 4, one right angle; theorem 5, two right angles; theorem 6, a right angle and a zero angle; theorem 7, one zero angle produce the particular conclusion. In the theorem of parallels cutting equal segments on a transversal, etc., three metrical premises produce the conclusion, two zero angles and a pair of equal segments.

In summation we have the following

CLASSROOM FACTORS.

What, dominant operation and metrical premises and conclusion.

Wherewith, the tool.

Construction, to use the tool, with declaration of immediate and ultimate purpose.

Consequences, the 3 F's, figures, facts, followings, the last consequence being the conclusion.

Diagrammatic designation of the metrical premises by heavy lines and middle letters.

Diagrammatic designation of the conclusion, the goal, by end letters.

Diagrammatic delineation of the march of the proof by initial letters in order.

With these factors clearly understood, the student cannot fail to *understand* the demonstration. Without these specific details the bulk of classroom work is mere parrot work.

PROBLEM DEPARTMENT.

PROFESSOR IRA M. DELONG,
University of Colorado, Boulder, Colo.

Readers of the Magazine are invited to send solutions of the problems in this department and also to propose problems in which they are interested. Solutions and problems will be duly credited to the author. Address all communications to Ira M. DeLong, Boulder, Colo.

ALGEBRA.

17. **Bacchus caught Silenus asleep by the side of a full cask, and seized the opportunity of drinking, which he continued for two thirds of the time in which Silenus would have emptied the full cask. After that Silenus awoke and drank what Bacchus left. Had they both drunk together the cask would have been emptied two hours sooner, and Bacchus would have drunk only half what he left Silenus. Required the time in which each could empty the cask separately.**

Solution by A. E. Hennings, M. A., Walla Walla, Wash.

Let B = No. hours it takes Bacchus to empty cask.

S = " " " " " Silenus " " "

$\frac{1}{B}$ = part of whole cask Bacchus drinks in one hour.

$\frac{1}{S}$ = " " " " " " " " "

$\frac{2S}{3}$ = No. hours Bacchus drank while Silenus was asleep.

$\frac{2S}{3} \times \frac{1}{B}$ or $\frac{2S}{3B}$ = part of whole cask Bacchus drank in this time.

$1 - \frac{2S}{3B}$ or $\frac{3B - 2S}{3B}$ = part of whole cask Bacchus left.

$\frac{3B - 2S}{3B} \div \frac{1}{S}$ or $\frac{BS - 2S^2}{3B}$ = No. hours Silenus drank.

Then, $\frac{2S}{3} + \frac{3BS - 2S^2}{3B} = \frac{2}{\frac{1}{B} + \frac{1}{S}} + 2 \ldots\ldots (1)$

$\frac{1}{2}\left(\frac{3B - 2S}{3B}\right)$ or $\frac{3B - 2S}{6B}$ = part Bacchus would have drunk, both drinking together.

$1 - \frac{3B - 2S}{6B}$ or $\frac{3B + 2S}{6B}$ = part Silenus would have drunk, both drinking together.

Then, $\frac{\frac{3B - 2S}{6B}}{\frac{3B + 2S}{6B}} = \frac{\frac{1}{B}}{\frac{1}{S}}$ or $\frac{3B - 2S}{3B + 2S} = \frac{S}{B} \ldots\ldots (2)$

From (2) $B = 2S$.

Substituting this value for B in (1) we have $B = 6$ hrs, $S = 3$ hrs.

Also solved by Ralph E. Root, E. L. Brown, I. L. Winckler, W. H. Hays, R. S. Beardsley. Two incorrect solutions were received.

18. *Proposed by H. C. Whitaker, Ph. D., Philadelphia, Pa.*

A board of uniform thickness is a feet wide at the narrow end and b feet wide at the broad end. It is desired to cut off one nth of the board by a cut parallel to the two ends. How far from the narrow end must the cut be made and what will be the length of the cut?

Solution by E. L. Brown, M. A., Denver, Colo.

Let ABCD be the board, AB $\equiv$ a, CD $\equiv$ b, RS the cut parallel to the two ends. Let CB and DA meet at P. Draw PN perpendicular to CD cutting AB in M, RS in L, CD in N. Let ML $\equiv$ X, RS $\equiv$ Y, and MN $\equiv$ l, the length of the board.

Now $\frac{PM}{l} = \frac{a}{b-a}$, whence $PM = \frac{al}{b-a}$.

Area $\triangle$ PAB $= \frac{a^2l}{2(b-a)}$; area ABSR $= \frac{l(a+b)}{2n}$.

Therefore area $\triangle$ PRS $= \frac{a^2l}{2(b-a)} + \frac{l(a+b)}{2n}$.

Because $\triangle$s PAB, PRS are similar, we have

$$\frac{Y^2}{a^2} = \frac{a^2l}{2(b-a)} + \frac{l(a+b)}{2n} \div \frac{a^2l}{2(b-a)}.$$

From this it is seen that $Y^2 = \frac{(n-1)a^2+b^2}{n}$, an expression independent of the length of the board.

We have also, $\frac{X}{2}(a+y) =$ area ABSR $= \frac{l(a+b)}{2n}$

$\therefore X = \frac{l(a+b)}{n(a+y)}$.

Solved with the same value for y by I. L. Winckler, Ralph E. Root, R. S. Beardsley, W. H. Hays, A. E. Hennings, Eleanora Harris, O. R. Sheldon, and the proposer.

A solution which gives y in terms of l, the length of the board, was received from P. G. Agnew.

The following Algebra solutions were received too late for crediting in the April issue: Ralph E. Root, Nos. 8, 10; W. H. Hays, No. 11.

TRIGONOMETRY.

21. Given $\cos 4\theta = -\frac{1}{2}$; find the general value of θ.

Solution by Ralph Root, Sc. B., Forest City, Ia.

$\theta = \frac{\pi n}{2} \pm 45^\circ \pm 15^\circ$ where n is an integer.

Solution by E. L. Brown and Eleanora Harris.

$(2n+\frac{1}{4})\pi \pm \frac{\pi}{12}$, $(2n+\frac{3}{4})\pi \pm \frac{\pi}{12}$,

$(2n+\frac{5}{4})\pi \pm \frac{\pi}{12}$, $(2n+\frac{7}{4})\pi \pm \frac{\pi}{12}$.

Solution by H. C. Whitaker, Ph. D., and I. L. Winckler.

$\theta = \frac{1}{6}(3n \pm 1)\pi$, where n is any integer.

Solution by R. S. Beardsley, Chicago, Ill.

$\theta = n \cdot 45°, \pm 15°$ where n is an odd integer.

Solution by W. H. Hays, Columbia, Mo.

$\theta = \frac{n\pi}{2} + \frac{\pi}{6}, \frac{n\pi}{2} + \frac{\pi}{3}$, where n is any integer.

One partially incorrect solution was received.

Solutions of Trigonometry No. 13 were received from R. A. Brown, W. H. Hays and Grace M. Bareis too late for crediting in the April issue.

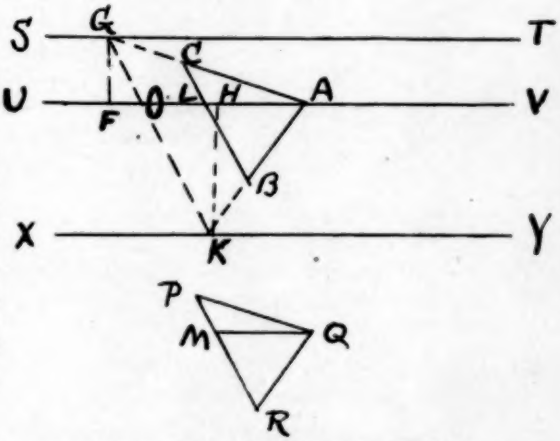
GEOMETRY.

19. *Proposed by P. F. Gaehr, Ithaca, N. Y.*

Construct an equilateral triangle whose vertices shall be on three given parallel lines.

Solution by *I. L. Winckler, Cleveland, O.*

Let PQR be the given triangle, and ST, UV, XY the given parallels. Let a and b represent the distances between ST and UV, and UV and XY, respectively.



Divide PR at M, into parts proportional to a and b, that is so that PM : MR = a : b.....(1)

Construct the triangle ABC = triangle PQR (the angles C, A, and B being equal respectively to P, Q, and R), so that A falls on UV and QM takes the position AL along UV.

Produce AC to meet ST at G and AB to meet XY at K. Draw GK cutting UV at R and draw GF and KH perpendicular to UV. Then GF : KH = GO : OK or a : b = GO : OK.....(2).

From (1) and (2) we have

GO : OK = PM : MR

or GO : OK = CL : LB (since CL=PM and LB = MR)

∴ GK is parallel to CB and the triangle AGK is similar to ABC and hence to the given triangle PQR.

(There will be three solutions when the triangle has the three sides unequal, obtained by dividing each side in turn into parts proportional to the distances between the parallels.)

Also solved by H. C. Whitaker, W. H. Hays, R. E. Root, R. S.

Beardsley, A. E. Hennings. A trigonometric solution was received from E. L. Brown.

20. *Proposed by W. D. Higdon, St. Louis, Mo.*

The chord of a segment is 10 feet and the radius is 16 feet. Find the area of the segment.

Solution by H. C. Whitaker, Ph. D., Philadelphia, Pa.

$$\text{Area Segment} = \text{Area Sector} - \text{Area Triangle}$$

$$= r^2 \arcsin \frac{C}{2r} - \frac{1}{4} C \sqrt{4r^2 - C^2}$$

$$= 81.3631 - 75.9936 = 5.3695$$

Also solved by R. S. Beardsley, R. E. Root, W. H. Hays, E. L. Brown. One incomplete solution was received.

The following Geometry solutions have been received: W. H. Hays, Nos. 10, 13, 15; R. E. Root, Nos. 13, 15; J. F. West, Nos. 13, 15; J. S. Brown, Nos. 13, 15.

PROBLEMS FOR SOLUTION.

ALGEBRA.

27. *Proposed by Eliza Robinson, Duluth, Minn.*

Two men start from opposite sides of a lake to row directly across. Their rates of rowing are uniform but not necessarily the same. They meet 720 rods from the left hand shore and each rows to the starting point of the other, rests ten minutes, and rows back again. This time they meet 440 rods from the right hand shore. Find the distance across the lake.

28. *Proposed by H. C. Whitaker, Ph. D., Philadelphia, Pa.*

A is one mile north of a certain point, B is one mile west. A travels due southeast and B travels east, both starting at the same time and moving at the same rate. How near will they approach each other?

GEOMETRY.

29. *Proposed by I. L. Winckler, Cleveland, O.*

Show how to bisect the area of any quadrilateral by a line passing through a given point in one of its sides.

30. *Proposed by P. G. Agnew, Washington, D. C.*

A frustrum of a cone has its radii R and r and its altitude h. Where will a plane parallel to the bases cut off $\frac{1}{n}$ of its volume?

TRIGONOMETRY.

31. *Proposed by E. L. Brown, M. A., Denver, Colo.*

In a triangle $\frac{\sin A}{\sin B} = \frac{m}{n}$, and $\frac{\cos A}{\cos B} = \frac{p}{q}$.

Show that $\cos C = \frac{mp - nq}{np - mq}$

AN AXIOM FOR THE STRAIGHT LINE.

By John James Quinn,

Warren, Pa., High School.

Due to the initiative and untiring energy of Dr. George Bruce Halsted, whose researches in non-Euclidean fields have exposed the underlying fallacies of the definitions and postulates relating to the straight line as set forth in many modern text-books of plane geomerty, the Mathematical Section of the Ohio Association of Teachers of Mathematics and Science have resolved that in teaching demonstrative geometry they will not use the phrase, "A straight line is the shortest distance or path between two points;" or the phrase: "The shortest distance between two points is measured on the straight line joining them." The reasons given for this action are:

1. That the idea of a minimum expressed by the term "shortest" is too complex in its ultimate analysis, and involves concepts not intended as assumptions.

2. That the definition of a straight line should be based upon its fundamental idea of quality rather than upon a specific property.

Finally the manner of defining a straight line with reference to its qualitative property is left to the judgment and ingenuity of the teacher as well as the task of formulating another axiom.

In texts which begin with the triangle it is an easy matter to avoid the use of this axiom by assuming that "one side of a triangle is less than the sum of the other two sides," but if they begin with the simple relations of lines and angles it seems necessary that some axiom be substituted for the one rejected. The following is suggested:

The broken line between two fixed points is longer than the straight line joining them.

This involves only the qualitative property of the straight line which the mind conceives in apprehending it, and no attention is drawn to any special property.

THE EQUATION OF A TANGENT TO A CURVE.

Though many methods have been given, the following, based upon my treatment of the angle-bisection theorems as given in School Science and Mathematics, may be of interest.

(a.) It considers the tangent as the ray of instantaneous motion.

(b.) It does not involve the theory of limits.

(c.) It does not require the equation of the curve (except to reduce it to the standard form).

1. In the parabola let $P \equiv (x', y')$, the focus $F \equiv (a, o)$. The equations of FP and the axis-parallel through P each being in normal form are

$$\frac{x'-a}{x'+a}y - \frac{y'}{x'+a}x + \frac{a}{x'+a}y' = 0 \quad (1)$$

$$y - y' = 0 \quad (2)$$

Adding

$2x'y - y'x - x'y' = 0$ is equation of tangent

As $x' = \frac{y'^2}{4a}$ $\qquad y\,y' = 2a\,(x + x')$

2. In the ellipse let $P \equiv (x', y')$, the foci $F \equiv (c, o)$, and $F' \equiv (-c, o)$ the equations of $F'P$ and FP in the normal form are

$$\frac{1}{r'}(x' + c)\,y - y'x - c\,y' = 0 \quad (1)$$

$$\frac{1}{r}(x' - c)\,y - y'x + c\,y' = 0 \quad (2)$$ Subtracting

$x'^2\,y - x\,x'\,y' - a^2\,y + a^2\,y' = 0$ is equation of tangent.

As $x'^2 = a^2 - \frac{a^2}{b^2}y'^2$ $\qquad \frac{x\,x'}{a^2} + \frac{y\,y'}{b^2} = 1$

Ames, Ia. T. M. BLAKSLEE.

AN EXPLANATION.

The mathematical editor wrote and intended to insert the following footnote to the article of last month by Mr. Floyd on the "Trisection of an Acute Angle."

This article was prepared by a high school boy and is published here exactly as his teacher sent it to the editor. It is a *bona fide* piece of work by a high school pupil and its crudities and errors are all included here. Of course, even high school teachers of mathematics know that the trisection of an acute angle has been proved impossible by means of the ungraduated ruler and the compasses alone. It will be a good exercise for some other high school boy to point out the reason for the failure of this method. The article will furnish the basis for profitable discussion by a secondary school mathematical club.

The editor's oversight was caused by the press of very heavy work at the time of preparing the May manuscripts for the printer.

Arrangements have been made for a brief article on the Trisection Problem, giving a summary of the history, literature and scientific *status* of this classical difficulty. MATHEMATICAL EDITOR.

DEPARTMENT OF METROLOGY. NOTES.

Littauer Bill not Reported.—The House Committee on Coinage, Weights, and Measures, after having closed its hearings for and against the metric system, has voted seven to five (there are seventeen members on the Committee) not to report the Littauer Bill at this session of Congress. R. P. W.

Circular of Appeal.—The following circular, issued by the American Metrological Society, merits careful reading and direct action on the part of every reader, and particularly every teacher of science or mathematics. The opponents of the Metric System are few, but they are as active as they are myopic, and they do not scruple as to the methods they employ. The friends of the system must work for it if they have any regard for the better education in arithmetic of the millions of children in the schools of to-morrow. R. P. W.

Metric Propaganda in Canada.—The Daily Consular and Trade Reports for March 16 contains a brief statement by Consul Seyfert of Stratford, of the active movement now in progress in Canada, in favor of the Metric System. He says:

"With a view of adopting the metric system in weights and measures at an early day the Canadian government is preparing and educating the people for the change. Prof. J. C. McLennan, of the University of Toronto, has been engaged by the Dominion government to devote the next year to explaining the system in a series of lectures in all the leading cities from Halifax to Vancouver. The professor delivered his first lecture on the subject at Stratford, February 7, before the board of trade. According to his explanation it is the object of the Canadian government to secure a uniform standard of weights and measures throughout the entire British Empire, and thus to advance trade relations among the different colonies of the Empire. The board of trade at Stratford unanimously adopted resolutions, urging adoption of the system."

Thus is being demonstrated month by month the world-wide movement in favor of educational and intellectual, no less than of moral and social reform. R. P. W.

THE ARTIFICIAL INDUCTION OF LEAF FORMATION IN THE OCOTILLO.

By F. E. Lloyd.

The Ocotillo is a desert plant which during the dry season bears no foliage leaves, following the first rains its thorny branches soon become covered with new leaves. The author of this article thought "that it would be instructive to see if any light could be obtained upon the influence of metoric water upon the development of leaves." Bandaging a portion of a branch with cheese cloth which was so connected with a reservoir of water in a bottle that it remained constantly moist. In three days leaves began to appear below the bandage for 12 to 15 inches. In case of this experiment the leaves did not grow so rapidly as they do when growing following a rain.—*Plant World, March, 1906.*

The Chemistry Teachers Club of New York City has undertaken a most practical and unique method of improving and enriching the course in chemistry in the New York High Schools. Each member sends to a committee any new idea or improvement in apparatus, or new ways of performing an experiment.

The committee selects such as are considered worthy of preservation. These are printed on cards of a suitable size for a filing case and a set furnished to each member of the Club.

The actual work of printing is done by one of the members, Mr. Henry T. Weed of the Manual Training High School, Brooklyn.

The Club has voted to give a set of these cards as they appear to School Science and Mathematics. Selections from them will be printed in future numbers.

REPORTS OF MEETINGS.

NEW JERSEY STATE SCIENCE TEACHERS' ASSOCIATION.

On April 28 the Science teachers of New Jersey met in Newark and organized a New Jersey State Science Teachers' Association. The following officers were elected: President, Mr. A. T. Seymour of Orange High School; vice-president, Mr. H. M. Campbell, of Long Branch High School; secretary-treasurer, Mr. Gilbert H. Trafton, of Passaic High School.

The executive committee consists of the officers and the following members: Miss Grace E. Cooley of Newark High School; Mr. F. M. LaFayette of Camden High School; Mr. W. Weiner of Newark High School; and Mr. C. C. Wilson of Jersey City High School.

A committee was appointed to confer with the committees from other similar organizations regarding a syllabus in physics.

G. H. TRAFTON.

CHEMISTRY TEACHERS' CLUB OF NEW YORK CITY.

THIRTEENTH MEETING, DECEMBER 9, 1905.

After the usual dinner at the Hotel Savoy the Club met in Room 201, Board of Education building.

The minutes of the last two meetings were read and approved. In the absence of Mr. Weed the committee on printing made no report. It was voted that that part of the minutes of the twelfth meeting in which the processes at the smelting plant were outlined should be submitted to the printing committee.

Upon the recommendation of the executive committee the following were elected members of the Club: H. S. Turner, Montclair High School; M. A. Rosanoff, New York University; C. H. Wagner, High School of Commerce; B. F. McFarland, New Haven, Conn.; R. B. Rodman, DeWitt Clinton High School; H. R. Wood, Flushing High School.

A communication from C. M. Turton, business manager of SCHOOL SCIENCE AND MATHEMATICS, was submitted to the committee on printing, with power to act. At the close of the business session the regular program of the evening was taken up.

Professor Fay reported that ozone was present in the oxygen liberated from HgO, Pb_3O_4, PbO_2, MnO_2 and $HClO_4$. "Starch-iodide" paper was used in testing. Mr. Easton obtained ozone from PbO_2, from water which contained anywhere from 1 to 20 per cent of sulphuric acid, and from MnO_2 in rather large amounts. In the oxygen liberated from hydrogen dioxide, rather curiously, no ozone was to be found.

In reporting upon the best proportions of potassium chlorate-manganese dioxide mixture for laboratory use, Mr. Currie stated that of the proportions suggested (viz., 10 of $KClO_3$ with from 1 up to 10 of MnO_2), he found 10 of $KClO_3$ by weight to about 11 of MnO_2 to be, on the whole, the best, taking into consideration both ease and regularity of evolution. Mr. Bright sent a written report, from which

the best proportions seemed to be 10 per cent $KClO_3$ to 9 or 10 per cent MnO_2.

Mr. Jaquish stated that while working with the $KClO_3$ and MnO_2 mixture, he had become interested in the formation of $KMnO_4$ as an intermediate product. He had found that 1 part of MnO_2 to 5 of $KClO_3$ gave him the best yield, while 5 per cent $KClO_3$ to 7 per cent MnO_2 gave the poorest. Mr. Wiener reported that 10 per cent $KClO_3$ to 8 per cent MnO_2 gave the best results for oxygen. Dr. Walker found that 10 per cent $KClO_3$ with 10 per cent MnO_2 gave off oxygen most uniformly, while 10 per cent $KClO_3$ to 4 per cent MnO_2 evolved oxygen most rapidly. For ordinary work the production seemed unimportant. Mr. Whitsit, using 10 parts by weight of $KClO_3$ in every case, made mixtures containing MnO_2 in proportion by weight of from one tenth up to 30. He found a 10 to 10 mixture gave off oxygen at the lowest temperature, but that mixtures containing from 15 to 25 parts by weight of MnO_2 gave off the oxygen with greatest rapidity at low temperatures.

In making the above tests, except the last, the test tubes were heated directly over the lamp or were imbedded in sand. In all cases efforts were made to obtain identical or comparable conditions. Mr. Whitsit using the comparison tube heated two tubes at a time and employed two special pieces of apparatus to obtain uniform heating, one a small "tinned" iron air bath asbestos covered, the other two melting point tubes hung in a large test tube which in turn was suspended in a heated glass flask.

The third topic for the evening had to do with electrolysis. Mr. Bennett reported that in the electrolysis of hydrochloric acid with the Hofmann apparatus carbon electrodes rapidly disintegrate and the results obtained are poor. With platinum electrodes, if the gas levels are adjusted after the solution is saturated with chlorine, the evolution of the two gases is quite uniform; if now the current be stopped and then started again the hydrogen gains on the chlorine. The solution used contained 160 grams NaCl, 140 H_2SO_4 and 700 of H_2O.

Using a special apparatus in which the cathode was surrounded by a porous cup diaphragm, he obtained 40 cc. of chlorine to 44 of hydrogen.

Professor Rosanoff showed Professor Loeb's modified Hofmann's apparatus for the electrolysis of brine. To avoid the mixing of the evolved gases, the carbon electrodes are inserted (with an upward slant) some distance above the U bend. To avoid the gain of the hydrogen over the chlorine due to the greater solubility of the latter under pressure, an outlet stopcock is placed part the way up the supply tube (at the back), by means of which outlet the pressure on the evolved gases can be reduced or even made negative.

Mr. Mills reported upon the electrolysis of fused sodium hydroxide. If the terminals from the street direct current lighting circuit, the current being cut down to about one ampere, be "arc"-ed upon a stick of sodium hydroxide and then separated, the alkali will fuse and a

bead of sodium will collect at the cathode. To show the oxygen and hydrogen, iron electrodes are enclosed in glass tubes which dip into molten sodium hydroxide, held in an iron dish and heated by a burner. With a current of 3 or 4 amperes, oxygen and hydrogen will quickly collect at the two electrodes.

Mr. Sohon illustrated the Castner process for the electrolysis of brine. A side armed U tube, with electrodes passing through stoppers, has an obstruction placed partly across the anode side at the beginning of the bend. The bend is then filled with mercury, a salt solution poured into the obstructed side, and very dilute NaOH upon the other. Upon passing the current, the chlorine escapes while the sodium forms an amalgam with the mercury. By tilting the tube, the amalgam can be transferred to the other side where it will react with the water. By rocking the tube the evolution of hydrogen and chlorine is made continuous.

Mr. Sohon also illustrated a quick method for determining the hydrogen equivalent of magnesium. A gas measuring tube containing a little concentrated HCl is filled with water in such a way as not to mix it with the acid, and a weighed piece of Mg held in position by a string, is placed in the water at the top of the tube. On now inverting in a bowl of water, the heavy HCl solution will mix with the water and react with the magnesium. The evolved hydrogen is measured, and the usual calculations made. Time required for the manipulation, exclusive of the weighing of the Mg, is less than ten minutes.

Professor Fay gave a qualitative demonstration of osmotic pressure. The osmotic cell was a porous cup coated inside with copper ferrocyanide (as directed by Morse of Johns Hopkins), filled with a sugar solution and placed in a battery jar of water. Sealed in the top of the porous cup was a long tube in which, floating on the sugar solution, was an index of kerosene colored with chlorophyl. During the course of the evening this index rose several feet. The great difficulty in the making of the cell lies in the preliminary cleaning of the porous cup.

After a short discussion of the experiments described the meeting adjourned. Jos. S. Mills, Secretary.

CHEMISTRY TEACHERS' CLUB OF NEW YORK CITY.

Fourteenth Meeting, March 30, 1906.

After an informal dinner at the Mohawk Hotel, Brooklyn, the Club met in the main lecture room of Pratt Institute. An unusually large number of members and guests were present.

Mr. Hancock reported that to the printing committee it seemed advisable to have the club's experiments and reports printed on catalogue cards; sample cards exhibiting headings, etc., were shown. After some questioning and discussion the entire matter was referred to the printing committee with power to act.

The following gentlemen were elected to membership. Allan Rogers,

and C. E. Bivins, of Pratt Institute, Brooklyn; Walter G. Whitman, Ethical Culture High School, New York; Oscar R. Flynn, High School of Commerce, New York.

Dr. Allen Rogers outlined rather fully the courses in chemistry at the Pratt Institute, both those now given and those projected, particular attention being paid to the industrial work. While all the courses will be helped by the opening of the new chemistry building, those along industrial and technical lines will be especially benefited.

Professor Edgar F. Smith of the University of Pennsylvania delivered a most interesting illustrated lecture on the "Isolation of Metallic Calcium." After giving an outline of the earlier methods for the electrical preparation of both the alkali and the alkali earth metals, he described in detail the process perfected at the University of Pennsylvania by which sticks of calcium of any desired length may be obtained. Samples of calcium and of calcium alloys were exhibited.

After a vote of thanks to Professor Smith, the Club adjourned to view the new laboratories of the institute.

Jos. S. Mills, *Secretary.*

CENTRAL IOWA ASSOCIATION OF SCIENCE AND MATHEMATICS TEACHERS.

Through the efforts of an organizing committee composed of F. E. Goodell, Miss Zulema Kostomlatsky, and Hilton Jones all of Des Moines, the teachers of Mathematics and Science of Central Iowa met at Des Moines May 4 and 5, and the following program was given:

May 4, 7:30 P. M. Inspection of laboratories and equipment of North High School.

8:15 P. M. General meeting in the Assembly Hall of North High School. Mr. Hilton Jones was appointed chairman and Mr. H. B. Smith, secretary.

Music was furnished by the North High Orchestra. An address of welcome was given by Superintendent W. O. Riddell of the West Des Moines Schools.

The advisability of organizing as a branch of the Central Association of Science and Mathematics Teachers with School Science and Mathematics as an official organ was set forth by Mr. Goodell, and met with universal approval.

The discussion and adoption of a constitution then followed. The organization is to be known as the Central Iowa Association of Science and Mathematics Teachers and its object shall be to promote the better teaching of science and mathematics. The following officers were elected for the ensuing year: President, F. E. Goodell, Des Moines; Vice President, J. L. Van Horn, Boone; Secretary-Treasurer, Frances Church, East Des Moines; Executive Committee, Miss Maria Roberts, Iowa State College, Ames; W. A. Merrill, Des Moines.

The address of the evening was given by Professor W. S. Hendrixson of the Iowa College, Grinnell, on "A Science in the Making."

The evening closed with an informal reception.

Saturday the following program was given at the West High School:

9:00 A. M. Inspection of Laboratories and Equipment of West High.

10:00 A. M. Section Meetings.

Science Section.—Natural Science Lecture Room.

1. The Use of the Lantern in Science Work, by Miss Grace Troutner of North Des Moines High School.

General Discussion led by Professor L. S. Ross of Drake University.

2. Problems of Field Work in National Science, by Miss Edith Keefer of East Des Moines High School.

General Discussion, led by Miss Zulema Kostamlatsky of West High School.

3. Laboratory Experiments in Physics, by Professor D. W. Morehouse of Drake University.

General Discussion, led by Mr. Lafayette Higgins of West High School.

4. Discussion of New Books in Science.

Mathematics Section.

1. Limitation in Mathematics, by Principal W. A. Crusinberry of West High School.

2. Rational Geometry, by Miss Blanche Pinkerton of Valley Junction.

3. The Relation of Physics and Mathematics, by Professor L. B. White of Des Moines College.

4. Round Table on Problems in Teaching Mathematics, led by Principal Crusinberry.

Great interest was shown in the meeting. The papers and the discussions were of much value. Before adjournment, another general assembly was held in which it was decided that meetings be semi-annual. The advantage of having these meetings in conjunction with those of the Iowa Academy of Sciences was discussed and the matter was left in the hands of the executive committee.

At 1:00 P. M., a demonstration of the Marconi Wireless Telegraphy was given at Our Circle Hall.

Our account of the lack of time, abstracts of papers cannot be given for this number of SCHOOL SCIENCE AND MATHEMATICS.

AMERICAN MATHEMATICAL SOCIETY.

The One Hundreth and Twenty-Eighth Regular Meeting of the Society was held in Room 305 of Schermerhorn Hall, Columbia University. The sessions began at 11:00 A. M. and 2:00 P. M. The Council met at 10:00 A. M. in Room 326 of University Hall.

In the interval between the sessions lunch was served as usual in the University restaurant.

The following papers were presented:

MORNING SESSION.

(1) Groups in which all the operators are contained in a series of subgroups such that any two have only identity in common. (By title.) Professor G. A. Miller, Stanford University.

(2) Lines of force illustrated by rotating carriage wheels.
Mr. W. H. Roever, Mass. Institute of Technology.

(3) Systems of lines of force whose differential equations take Bernoulli's form in polar coordinates
Mr. W. H. Roever.

(4) On twisted curves contained in a linear complex.
Professor Virgil Snyder, Cornell University.

(5) The number of classes of binary quadratic forms and ideals.
Mr. G. E. Wahlin, Yale University.
(Introduced by Professor Pierpont.)

(6) On the fundamental theorem in the reduction of multiple integrals.
Mr. R. G. D. Richardson, Yale University.

AFTERNOON SESSION.

(7) The notion of area of curved surfaces.
Professor James Pierpont, University of Missouri.

(8) Functions and their derivatives on given assemblages.
Professor E. R. Hedrick, Yale University.

(9) Lipschitz's condition in the case of implicit functions.
Professor E. R. Hedrick.
Professor Max Mason, Yale University.

(10) A necessary condition for an extremum of a double integral.

(11) An invariant of the calculus of variations corresponding to geodesic curvature. Professor G. A. Bliss, Princeton University.

(12) A generalization of conformal representation
Dr. Edward Kasner, Columbia University.

(13) Velocity curves in the dynamics of a particle
Dr. Edward Kasner.

(14) On a generalization of a problem of Tchebychev.
Professor J. W. Young, Princeton University.

(15) Concerning the bond uniting elements into a space.
Professor C. J. Keyser, Columbia University.

(16) Note on the differential invariants of a plane.
Dr. C. N. Haskins, Cornell University.

(17) On twisted quintic scrolls
Mr. E. C. Colpitts, Cornell University.
(Introduced by Professor Snyder.)

E. N. COLE, *Secretary.*

THE INDIANA ASSOCIATION OF SCIENCE AND MATHEMATICS TEACHERS.

The eleventh annual meeting of the Indiana Association of Science and Mathematics Teachers was held at Indianapolis April 28th. Previous meetings have covered Friday, Friday evening, and Saturday forenoon. On this occasion, however, it was thought best to confine the work of the meeting to one day. This was done in view of the fact that a large number of science and mathematics teachers in the state are not able to leave their work for the Friday sessions, but can be present with less inconvenience on Saturday.

The plan worked well in some ways, but it was attended by the difficulty of crowding the work too much, and thus making the day a very strenuous one for those who were present.

The session Saturday forenoon was general, and was held in the auditorium of the Manual Training High School.

The president's address on the subject, "Present Tendencies in Science Teachings," was delivered by Lynn B. McMullen, Instructor in Physics in the Shortridge High School. He said, there are three tendencies to be avoided: (1) The tendency to make science easy at any cost; (2) the tendency toward coddling the individual, and (3) the tendency toward extreme practicality. Over against these tendencies he suggested that we place that of doing a little science well, remembering that while curiousity may be aroused by skimming, true interest follows, and does not precede work.

This address was followed by a paper on "Mathematics below the High School," by Prof. O. L. Kelso, head of the mathematics department of the State Normal School. The main theme was a comparison of concrete mathematics with that which deals with pure number work. The speaker regards the most beautiful part of mathematics as that which has no relation to everyday life, and that elementary arithmetic is the quantitative side of human experience.

This paper was followed by an address on "What is Worth While in Biology," by Dr. D. W. Dennis, of Earlham College. The speaker stated that he always feels like taking off his hat to the species that survive by their persistence, many have fallen by the way as testified to by the rocks, and from these which have survived the teacher should draw a most striking lesson of the adaptation of himself to his environments.

The flowers and birds should be studied, for these show the greatest plasticity of adaptation. In teaching biology, he said, those subjects that are of a *popular* kind should be studied, such as the mosquito and its relation to yellow fever; or of a financial character as was studied and developed so thoroughly by Pasteur.

The literature and poetry of science should also be studied. Poetry will live long after accuracy has been buried in the depth of the deepest sea.

Science also should have a human interest, and thus be of more value to all who undertake its pursuit.

Following this "Our Weather" was discussed by Mr. W. T. Blythe of the weather department at Indianapolis. Mr. Blythe divides weather forecasters into three classes, (1) those of the "Ground-hog Day," goose-bone, or thick-hickory-nut-shell kind; (2) those who may be called the long-range, or year-in-advance kind; (3) those who base their calculations on strictly scientific observation, and only for twenty-four to thirty-six hours in advance. This last class compose the official forecasters of the weather service of the United States.

It is a welcome fact that the faith of the general public is being strengthened from year to year in the last class, while the first two

classes are losing ground rapidly with the progress of scientific investigations.

A short business meeting was now held, at the close of which the following officers were elected: President, Leonard Young, Evansville; Vice President, D. A. Rothrock, Indiana University, Bloomington; Secretary-Treasurer, John F. Thompson, Richmond; Executive Committee, O. W. Douglas, Chairman, Anderson; E. I. Kizer, South Bend.

In the afternoon the association met in section meetings, in adjacent rooms of the Shortridge High School.

The following sections were carried on simultaneously: Biology Section, Milo H. Stewart, Chairman; Chemistry Section, Frank B. Wade, Chairman; Physics Section, A. L. Foley, Chairman; Mathematics Section, D. A. Rothrock, Chairman.

Many papers of a most interesting and practical value were presented in these meetings. Each paper with its discussion was allowed eighteen minutes. An intermission of two minutes followed, so that the members might be permitted to change from one section to another.

The membership of the association was very materially increased, and quite an impetus was given to the already existing scientific spirit of those who attended the meeting. W. A. FISKE.

Dr. Saul Epsteen, formerly instructor, has been promoted to an assistant professorship in mathematics in the University of Colorado, being now second in rank to the head of the department.

BOOK REVIEWS.

Statistical Atlas of the Twelfth Census. Prepared under the supervision of Henry Gannett, Geographer of the Twelfth Census. Washington: U. S. Census Office, 1903.

This volume with 207 plates of maps and diagrams and 33 pages of descriptive matter is indispensable to the advanced student of geography. It should be in every high school library in the country, and available to every special student of geography. C. E. P.

Geology. By Thomas C. Chamberlin and Rollin D. Salisbury. Volumes II and III. New York, Henry Holt & Co., 1906.

These volumes contain the remarkable results of Professor Chamberlin's studies during the past decade or so, which would justify the application to the science they represent of the term, "The New Geology." They present matter of such fundamental importance that it seems best to call the attention of the reader to them at once, and to review them later. No student of geology or geography can be considered up with the times who has not read at least the portions of these volumes given to a consideration of the origin of the earth, the causes of climatic changes in the earth's history and expansional and restrictional evolutions of life. The work is designed as a text-book and not as a manual, and emphasis is placed on those portions which need greatest emphasis in a text-book. C. E. P.

Laboratory Course in Physical Measurements. A students' manual; revised edition. By Wallace C. Sabine, Professor of Physics in Harvard University. 8vo. Cloth. 97 pages. Ginn and Company, Boston. List price, $1.25; mailing price, $1.35.

This course, as given in Harvard College, is accompanied by a series of illustrated lectures to the class as a whole, one each week, and one hour a week of lectures, recitation, and problem work to the class in small sections. A knowledge of algebra and plane geometry, and a slight acquaintance with the notation of trigonometry are necessary. It should properly be preceded also by a more elementary course in physics, either by laboratory or text-book.

This manual, intended for students' use, has been given the form of an abstract of the daily lectures preceding the laboratory work and describing the experiments to be performed. For a number of experiments blue-print directions in regard to special points are placed beside the apparatus. The directions are on the whole general, and such details as depend on the special form of the apparatus are either left to the students' common sense or, when necessary, to special instruction.

Introduction to General Inorganic Chemistry. By Alexander Smith, Professor of Chemistry and Director of General and Physical Chemistry in the University of Chicago. New York, The Century Co., 1906, pages xviii + 780.

This new text, by one of the ablest teachers of chemistry in the country, is intended primarily as a college text. It will, however, be found invaluable to the teacher of chemistry in the secondary school, as a reference work. In its thoroughly up-to-date application of the principles of physical chemistry to general chemistry and in its admirable presentation of the fundamental laws of the science, it cannot fail to help the teacher who uses it, even though he be well grounded in the theory of chemistry. For those whose preparation was acquired before physical chemistry reached its present development this exposition and application of physical chemistry will be found well suited. The many paragraphs in fine print intended solely for the teacher or advanced student and still another feature of value to those who use the book for reference.

As to the character of the book as a text, the subject matter is presented in the historico-systematic method, plenty of acquaintance with materials employed being given before theoretical considerations are attempted. The development of the matter of atomic weights, for example, follows the historical order, the pupil being taught first, combining weights from a large number of experimental data, then atomic weights are presented, the discrepancy between certain of them and the corresponding combining weights being pointed out, and a promise of future explanation made, which is fulfilled at the time value is discussed. This order, while not the simplest from the theoretical standpoint, is that in which the development came to the race.

and, as Oswald and many other students of psychology believe, it may be the order best adapted to reception by the individual.

Throughout the book this excellence of pedagogical method may be observed. In clearness and fulness of presentation and explanation, in adequate experimental foundation for the theoretical superstructure, in full and clear exposition of the necessary physical phenomena in each case, and in the frequent application of what has been taught, as opportunity offers, the work is a model of applied pedagogy.

FRANK B. WADE,
Chemical Department, Shortridge High School, Indianapolis, Ind.

The Elements of Physics. By S. E. Coleman, head of the Science Department and teacher of Physics in the Oakland, Cal., High School. 12mo. 458 pages; 340 illustrations. Price, $1.25. D. C. Heath & Co., Publishers.

With its descriptive Text, Illustrations, Experiments, and Problems, this book is a complete text-book for a year's course in Physics.

Coleman's "Elements of Physics" permits considerable elasticity in the amount of work done and adaptability to facilities, time and the teacher's plan. The book presupposes a knowledge of elementary algebra and plane geometry, but requires no further mathematical training. The subject matter has been selected primarily for its value as a part of general education, and includes an unusual amount of information based upon the facts of daily experience, introduced as illustrations and applications of physical principles.

The sequence of topics has been planned with the view of presenting the serious difficulties of the subject gradually and in the easiest order. This is especially true in Mechanics, where this purpose has led to a wide departure from the conventional treatment, which introduces a maze of abstractions at the outset in connection with Newton's Laws of Motion. Now in this work, Newton's Laws are presented not under "Matter" but under "Motion," and find their plan following Statics of Solids and Dynamics; i. e. after consideration of "Force" and "Motion." A well connected, logical sequence of ideas is further secured by considering the subject of Energy as a central thought. The development of topics about this main principle gives the text coherence and unity.

The Inductive and Deductive methods are both strongly developed. The first includes the discussion and interpretation of experimental data. The deductive method is emphasized in the study of the connection of laws nad principles, in their applications in the useful arts and in the explanation of natural phenomena, both in the body of the text and in the problems. Each section is followed by problems which immediately test the student's mastery of principles and laws by compelling their practical application. In addition to numerical examples, these problems present a wide range of well-known facts and demonstrate the governing laws of many familiar appliances.

The "Elements of Physics" marks a distinct advance in the choice

and arrangement of material; in its accuracy, the clearness of its style, the systematic co-ordination of related ideas, in the presentation of topics in connected sequence, in its observance of the scientific spirit and methods; in the treatment of Energy throughout the book; and in the prominence given in the text and in the problems to the philosophy of common things.

An Elementary Laboratory Course in Chemistry. By Frank B. Kenrick and Ralph E. De Lury. 18x25 cm. pp. 90. Toronto: Morgan and Co., 1905. Price $1.00.

This new laboratory outline deviates quite markedly from numerous other introductory works which bear somewhat similar titles. The keynote of the book is sounded in the introduction. It is stated that "all reference to the atomic and molecular theories has been avoided, not because the authors are prejudiced against these excellent hypotheses, but in order to counteract a prejudice in the opposite direction on the part of students. A large number of students still continue to come to the university with their minds simply stuffed with atoms and molecules, which they take to be the fundamental facts of Chemistry." This is certainly a grave fault. It is a singular fact, however, that those who are most careful to avoid any suggestion of the atomic and molecular hypotheses, will allow themselves to speak fearlessly about the ionic hypotheses, as if it were not, in reality, an extension of the two hypotheses, and as if it could be applied in its proper relationships, when these two hypotheses form no part, or a confused part, of a student's mental make-up. It might be just as grievous to have a set of students whose minds were stuffed with ions under such a limitation of view.

The experiments selected are all very excellent. Many so-called quantitative experiments are quite adequately and minutely described, and set forth with such clearness that they could be carried out with success by the average student. There can be no question of the usefulness of "quantitative" experiments, in moderate doses. Perhaps the authors have placed too much emphasis upon these quantitative experiments. They say, "When *purely quantitative* experiments have been introduced, it has been done largely with a view of relieving the overcrowding at a too limited number of balances." In addition to the experiments chosen to illustrate the fundamental laws of mass, there are experiments designed to lead the student to understand the various terms and manipulations used in chemistry. Thus, in part I, a few of the experiments are: Solution and Crystallization; Separation by Solution; Experiments on Air; Combustion of Magnesium; Chemical Substances; The Composition of Water; Weight of Hydrogen evolved when Magnesium dissolves in an acid; The Law of Reacting Weights; The Properties and Composition of Acids; Acids, Bases, and Salts, etc. A very prominent place is given to experiments which illustrate the newer views concerning solution, the law of mass action, reversible reactions, electrolytic dissociation, etc.

The authors have given us a novel form of text-book for the laboratory, which has one great merit: viz., a few well chosen experiments have served as a nucleus from which, by way of explanation and extension, the fundamental conceptions of the science of chemistry may be developed. The method employed will arouse the student to do some original thinking, and, at the same time, will give him opportunity to think in the laboratory, at a time when his experiment is in progress. The usual laboratory course is crowded with a multiplicity of experiments which use up the student's time, without leading him to anything new except the observation of properties and the performing of tests, many of which could be, and usually are, seen on the lecture table.

The authors have been unfortunate in trying to meet a local demand for a single course which may be administered at the same time to students who wish to gain only a general view of the subject of chemistry, and to those who intend to pursue the study of chemistry further. The outline would serve admirably as a culture course. A question of grave doubt as to its sufficiency arises when it is considered as a basis for the student who expects to continue his work in chemistry. Would it give him what he needs in the way of an adequate fund of facts as well as theories?

Fully one fourth of the book is devoted to an outline of qualitative analysis "for the benefit of those students who are so unfortunate as to have to pass an examination in the subject." An appendix is given in which the authors venture to use the terms atoms, molecules, valency. Further discussion is given of such subjects as Reversible Reactions, Rate of Reaction, Oxidation and Reduction, The Mass Law, Extent of Dissociation, Experimental Error, Glass-blowing.

The work is recommended to all who are interested in the improvement of the present methods of laboratory instruction in chemistry, which, as every one knows, are far from the ideal. LAUDER W. JONES.

The Elements of Geometry. By Walter N. Bush, Principal Polytechnic High School, San Francisco, and John B. Clarke, Department of Mathemetics, Polytechnic High School, San Francisco. Silver, Burdett & Co., New York, 1905, Pp. xii + 350.

Elementary Algebra. By Walter R. Marsh, Head-Master Pingry School, Elizabeth, N. J. Charles Scribner's Sons, New York. 1905, pp. vii + 395.

The Art of Geometry. By Arthur Latham Baker, Manual Training High School, Brooklyn. Sibley & Co., N. Y. 1905, pp. v + 48.

This book is open to the objection of too much definition—haggling before business begins.

Twelve compact pages is the extent of it here. To their credit, be it said, the definitions are well-worded and compact. But what profit a first year pupil can get from a question like: "What is the

test of a complete definition?" p. 3, Ex. 1—no matter how much talk about the abstraction has preceded—is very difficult to see. Fortunately there isn't much of this type of question.

An excellent feature of the book is the grouping of propositions with reference to methods of proof. There are, "The Group on Adjacent and Vertical Angles," "The Parallel Group," "The (2n—4) Right Angles Group," etc. The subject-matter of plane geometry is very fully grouped off in this way. Pupils will thus be led to recognize simplicity of procedure in difficult demonstrations and to get something of an insight into geometrical methods and procedure, rather than merely to hold in his mind some unnecessary sequence of individual propositions. The frequent summaries of the work of the different groups gives the pupil frequent opportunity to "round-up" what he has learned and to feel the satisfaction and encouragement that come from the consciousness of having "made a point," of having *mastered something*. It is these frequent successes that go far to brighten up the none-too-cheerful mathematical pathway. The writer does not believe photographic reproductions of actual models of the figures of solid geometry should be carried very far in a text-book—not as far as this book carries them. The faculty we must develop is that of discerning the relations for one's self, without having them first *exhibited*. This book does not use the crutch so long as another geometry nowadays urged upon high school teachers. Still, it uses it too much and too indiscriminately. Many figures not at all difficult for the pupil to image, are "thought out for him by a model." Not so much mere clearness, as vividness and initiatory power should be our aim in high school geometry.

The writer protests against a statement such as, "It will assist the pupil to remember the sequence of propositions of Group iv, etc," page 34. A pupil ought not to have to burden his memory with any particular author's sequence. He should be taught to justify himself from fundamentals. *Authority* has no place in geometry, but reason has great place.

In conclusion, the writer must say that this book stands for more real thinking in the right direction than is the case with any other text of high school geometry he has seen for ten years.

The surest guaranty of class interest in algebra is the early introduction and continuous use of the equation. The surest way to miss interest is to postpone the equation, and keep the class on a continuous stretch of definition and introduction. The algebra cited above has 18 pp. of Introduction and Definitions, the definitions lacking nothing in clearness, followed by 28 more pages on the operations applied to algebraic number, before the equation is touched. The long suspense for the pupil is decidedly objectionable. Too much getting ready and too little of the thing for which the readiness is sought is unnatural and unpedagogical.

The diction of questions and problems is simple, and clear, but the treatment of topics is distinctly formal. The book is easier than the

common run of texts, due conjointly to simplicity of statement, lucidity of arrangement, and excellence of typography. The problem-work is as formal and as soundly dead as is customary.

Graphs are introduced on page 158 and used to illuminate the solution of simultaneous linear equations. They are used again in connection with quadratic equations. In neither case does the treatment seek to anticipate analytical geometry. A brief chapter on logarithms could have been made more serviceable by lengthening it by the space given to permutations and combinations and to the progressions and by putting it in earlier. Some physics problems are put into the chapter on ratio and proportion. Aside from this and some good work with the graph the book is essentially an unusually well-written old-line book, and has before it a good audience among the most conservative high school teachers. The answers to problems are bound separately and furnished to teachers without extra charge.

The third of the books cited undertakes to give the pupil a guide—an unerring guide—as to geometrical procedure. The author (p. iii) makes the rather presumptuous claim: "The following pages * * * will provide the student such a course of procedure (technique) that he could not miss it if he would." If this were actually done it would deprive geometry of about all the value it has educationally and reduce it to mere mechanics. *To err* is human, to go absolutely and inevitably right is machine-like. But the author does not deprive the subject of geometry thus of its rational quality. He does, however, in what we may call this "bird's-eye view" of the subject, give many suggestions that will be helpful in teaching. It is a *teacher's* study of geometry and, in the writer's view, not at all a *pupil's* study of the science. The book is hereby commended to teachers of geometry in any part of our public, or private, elementary, or high school curricula. It will repay study. G. W. M.

Commercial Geography. By Henry Gannett, Carl L. Garrison, and Edwin J. Houston. New York: American Book Company, 1905.

Two of the main requirements in a text-book are, first, that it shall be trustworthy, and second, that the subject shall be presented from the right point of view and in such a manner as to appeal to the student, arouse his interest, and cause him to bring to bear upon its problems his best efforts and his store of knowledge.

So far as the trustworthiness of this book is concerned the names on the title page are a sufficient guarantee. The final test of the second requirement is to put the book in the hands of the student. If it stands the test, it may be pronounced a good book. The reviewer has not made this test, and criticisms of this book are therefore not final, so far as he is concerned, but an expression merely of his present judgment.

The book is made up of three parts beside an introductory chapter and a few pages of statistical tables. Part I, Commercial Conditions,

contains 74 pages; Part II, Commercial Products, 81 pages; Part III, Commercial Countries, 248 pages.

PART I. It is probably true that nearly everyone who has taught both physical geography and commercial geography is an ardent advocate of the close correlation of the two subjects. The reviewer is firmly of the opinion that a study of physical geography that does not include an extensive application of its principles to the geography of the world loses a large part of its practical value to the student. In the study of plains, plateaus, and mountains, if the great examples of these topographic features in the different continents are not so studied as to build up a picture of the great features of the continents in the mind of the student, a great opportunity is lost. Indeed a full appreciation of the climatic features of the lands and the relation of the belts of vegetation to these features cannot be obtained without such knowledge. Since there is ordinarily taught in physical geography the series of relationships beginning with that of temperature distribution to the spherical form of the rotating earth and ending with the relation of the distribution of life on the earth to the more complex determining influences, it would seem a pity not to extend the study of these relationships to man, and to lead the student, as far as his interest will permit, to study those aspecs of commercial geography in which the physiographic control is most conspicuous. Elementary physical geography is commonly taught in the first year of the high school at a time when many students are thinking of the choice of their lifework, and when a general survey of the great field of human activity represented in commercial geography is most opportune. It comes at a time when the students' interests are such, perhaps, that a detailed study cannot be most profitably made, but certain great pictures can be acquired and a knowledge of certain great relations, while deeper study may await further growth of the student.

It is evident that the authors of this book recognize the importance of approaching the study with a knowledge of the principles of physical geography, for a large portion of Part I on commercial conditions is given up to subjects ordinarily taught in physical geography. For the young student who has not had a course in physical geography the treatment is hardly full enough for him to acquire the great mental pictures that he needs, but for the student who has had such a course, this part will be a ready means for a review.

PART II. After a brief discussion of soils, the second part deals with products, classified as vegetable, animal, and mineral. The localities throughout the world where these various products are obtained are shown by a very valuable series of maps together with a diagrammatic representation of the percentage of the total each country produces. By the use of the tables of statistics for the United States at the end of the book, the amount for any country can be roughly calculated from each percentage diagram, provided the United States is represented in the diagram. A valuable feature in this part is the brief description of the products manufactured from the raw material, and the processes of manufacture.

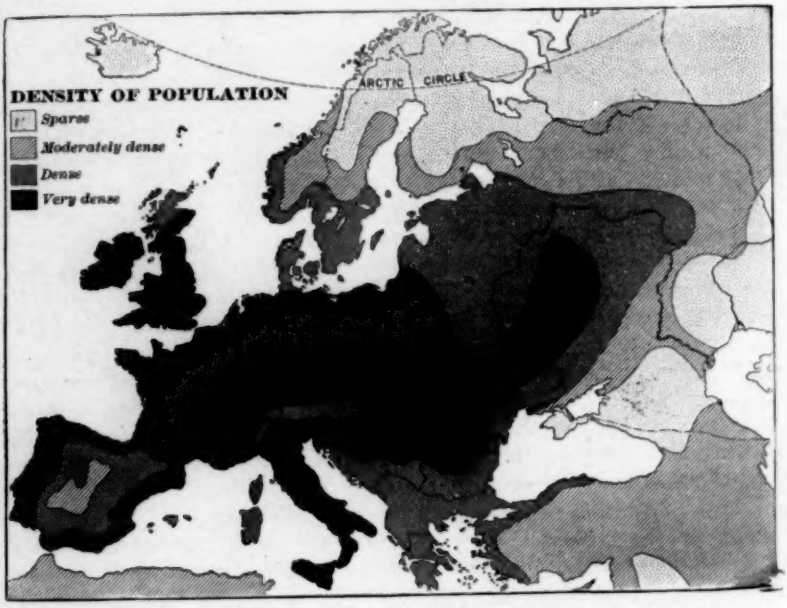
PART III. In this part the different countries are treated, commonly including their surface features, climate, vegetable, animal, and mineral products, manufactures, transportation, and commerce.

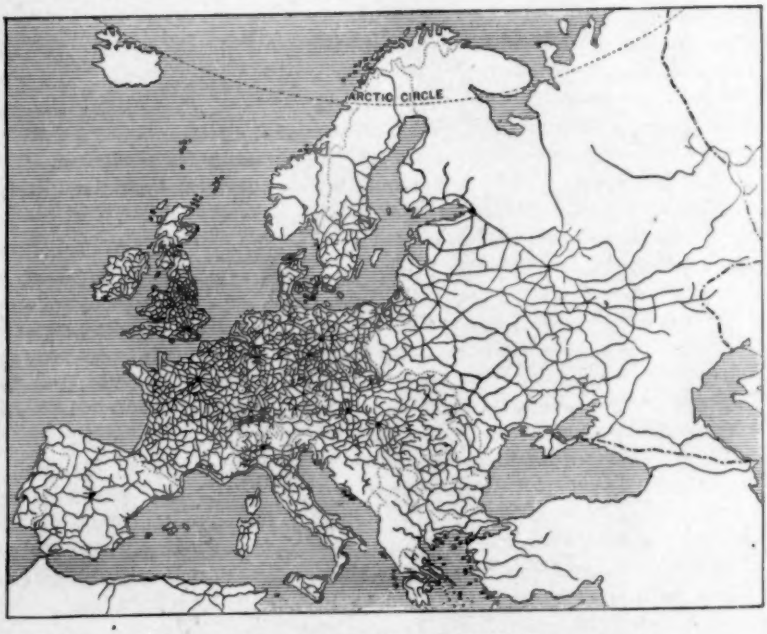
Again this part is full of valuable maps and diagrams together with a number of illustrative pictures. This is conspicuously true of that part dealing with the United States. A commendable feature of the maps is that (with one exception, so far as the reviewer has discovered) they contain the names of the places mentioned in the text, and not so many others as to make the maps confusing.

From the profusion of maps a few that show significant relations have been selected to accompany this review. We would direct attention to the significant relation between the great grain and live stock producing area of the country to the situation of Chicago with its great grain trade, packing houses, leather industry and superior transportation facilities. Other selections of maps are just as significant in relation to Chicago's lumber trade and furniture and iron manufacturing industries might have been made from the wealth of maps the book contains.

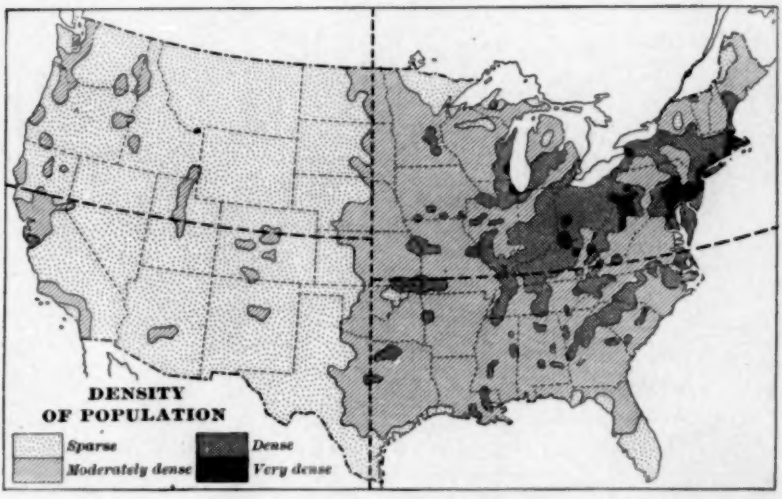
Those who had hoped for a reduction of memory work to a minimum by the presentation of great pictures, and a maximum use of the relation between the physiographic features and man's activities will be disappointed in this part of the book. In the treatment of Europe, while some of the countries are grouped together and treated as a whole, thus saving repetition in descriptions of regions naturally related, though separated by political boundaries, such boundaries nevertheless are permitted in many cases to separate the descriptions of regions which naturally belong together, so that the picture of these regions which would come with the greatest economy in one great whole is chopped up into several parts with no direct hint that the parts should be put together. Of course a good teacher will lead the student to put these parts together and to build up a picture of the naturally related regions in his mind's eye, but this might not unfairly have been expected of the text.

It is not alone in the treatment of the countries of Europe that the reader will be disappointed in the use made of the great pictures the student is supposed to have obtained from his study of Part I. To illustrate we would mention the following instances, not because they are the most important but because they show most easily what is meant. In speaking of the forest of Canada, there is no hint that it is a part of that great forest belt which practically encircles the earth, and which was once more extensive in North America and Europe before man cleared away so much of the forest. Again in speaking of furs there is no hint of the importance of this great forest belt both past and present, in the world's supply of furs, but reference is made to the political units from which the furs come. Again in treating of agricultural and grazing industries, it seems as if use might have been made of the knowledge the student is supposed to have acquired in Part I as to the grass lands, by using the terms there used. It would have been well also to show on the map of vegetation belts the agri-

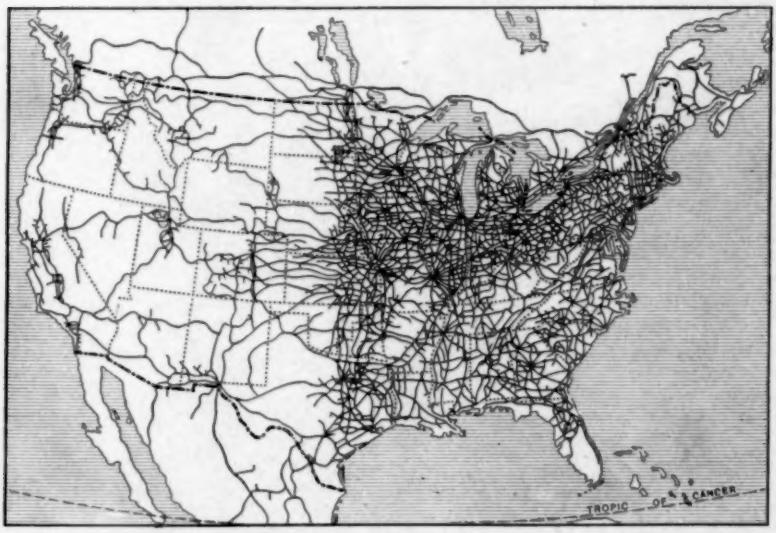




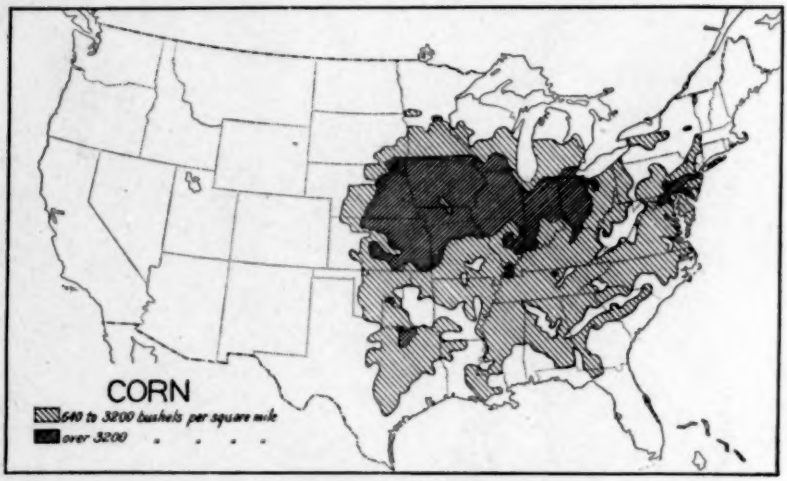
DENSITY OF RAILROADS IN EUROPE.



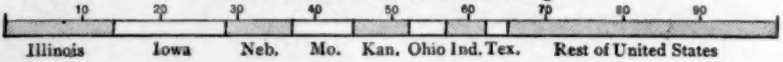
cultural lands, once forested, and to call attention in the test to their importance. Because of the scant use of the mental picures acquired from the physiographic descriptions and the failure to make maximums one of the physiographic relations the reviewer is almost ready to say that these descriptions simply make an additional burden for the student instead of saving his nrevous energy. This statement is perhaps not quite fair, and it seems almost ungrateful to complain because



DENSITY OF RAILROADS IN THE UNITED STATES.



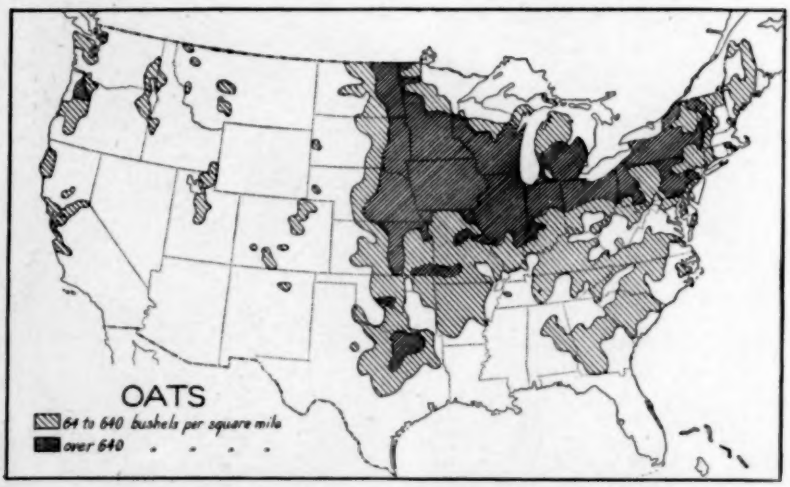
Indian Corn Production—Percentage



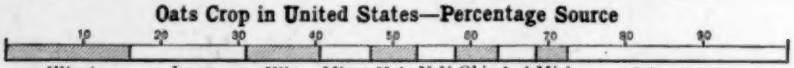
Illinois Iowa Neb. Mo. Kan. Ohio Ind. Tex. Rest of United States

the authors have not gone farther, when they have given more perhaps than any other American work. The material is in the book, and the good teacher will bring out the relations where the text does not.

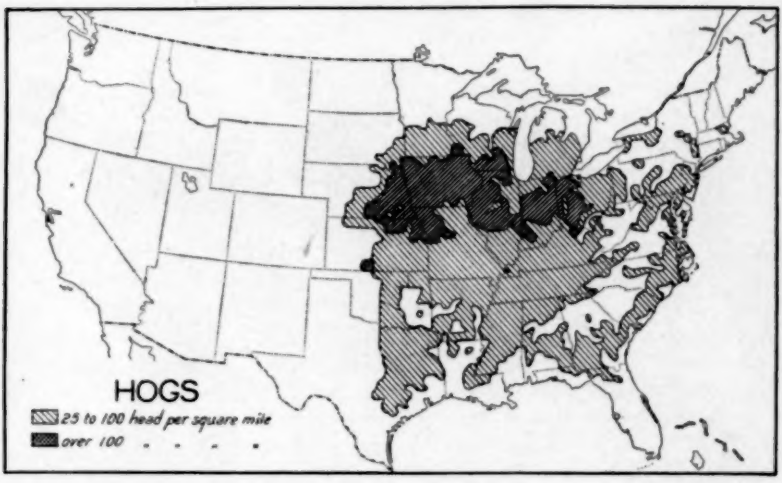
In general the language is clear. Some of the statements, however,



Oats Crop in United States—Percentage Source

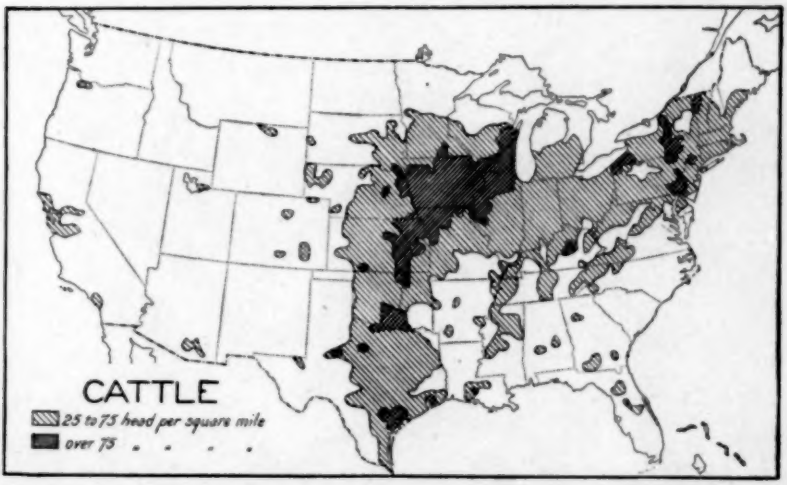


Illinois Iowa Wis. Minn. Neb. N.Y. Ohio Ind. Mich. Other States

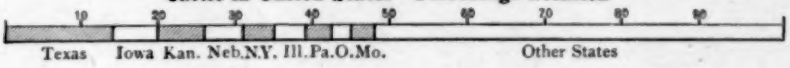


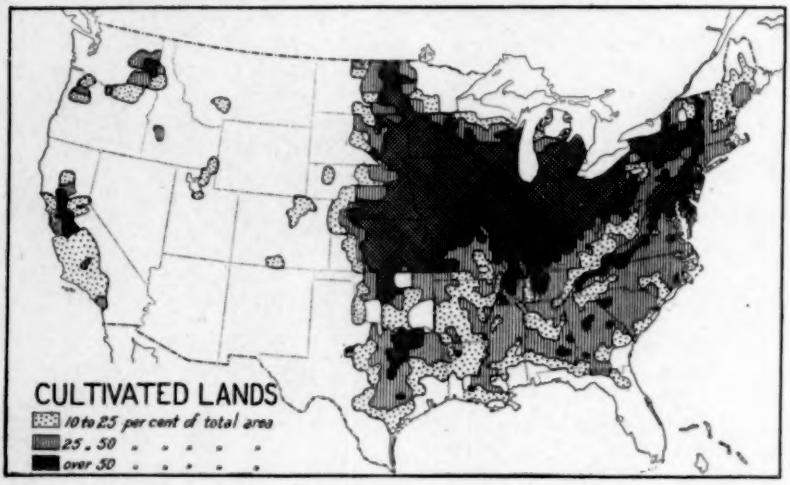
will leave the student in the dark. For instance, in speaking of Greenland, the statement is made that "cryolite, a mineral product, is found at Ivigtut" but the student is left, so far as the index shows, without any resource in this book to find what use is made of the mineral cryolite. To be sure he can go to the dictionary, but a word or two additional would save him this trouble.

Again, in speaking of the mineral products of Canada, the statement is made, "Nickel exists in large quantities, Canada supplying about one

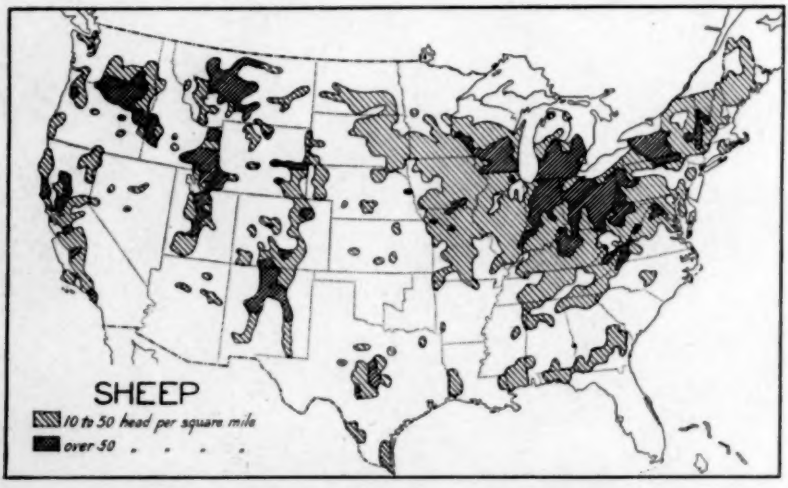


Cattle in United States—Percentage Location





half the world's product." No statement is made as to where the Canadian nickel is found. Although it is shown on the map a few pages back, no reference is made to this map. Indeed all through the book there is altogether too little reference to the maps and figures. Since a large part of the other half of the world's nickel is obtained in New Caledonia and Siberia, it seems as if it would have been worth while stating what rivals the Canadian mines have. In no one place have we been able to find a statement as to the production of the nickel of the



Sheep in United States—Percentage Location

Montana Wyo. N.Mex. Idaho Utah Ore. Mich. O. Other States

world. It is referred to in three places, although the principal source, the Canadian mines, would never be known if one relied on the index for finding it.

There are a few slips in accuracy of statement. On page 21 the idea is given that metamorphic rocks are those "originally of aqueous origin," thus implying that there are no metamorphic igneous rocks. On page 155 the origin of petroleum is assigned to "the decomposition of vegetable matter," apparently to the exclusion of animal matter. On page 156 the statement is made: "The United States produces about 50 per cent of the world's supply of petroleum, and Russia a little less," while on page 199 "the production of petroleum in southern Russia is greater than in the United States."

To mention such errors seems trivial, but it must be remembered that this book will be used by thousands of young people, and that the best is none too good for them, so that anything which author, publisher, or teacher can do to perfect the work should be done. To find so few errors in a work with the vast amount of detail it contains should strengthen confidence in it. Altogether in spite of the criticisms that have been made it is believed that teacher and student should be grateful to the authors and publishers for this work.

CHARLES EMERSON PEET.

BOOKS RECEIVED.

AMERICAN BOOK COMPANY, CHICAGO.

Brooks & Hubbard's Composition-Rhetoric. By Stratton D. Brooks, Superintendent of Schools, Cleveland, Ohio, and Marietta Hubbard, formerly of English Department, High School, La Salle, Illinois. Cloth, 12mo, 442 pages. Price, $1.00.

Smiley's Manual of American Literature. By James B. Smiley, A.M., Assistant Principal of Lincoln High School, Cleveland, Ohio. Cloth, 16mo, 336 pages, with portraits. Price, 60 cents.

Baldwin's Thirty More Famous Stories Retold. By James Baldwin, author of Balwin's Readers, etc. Cloth, 12mo, 235 pages, with illustrations. Price, 50 cents.

Theory of Functions of Real Variables. By James Pierpont, Yale University. 560 pages; price $4.50. Ginn & Co.

Elementary Algebra. By G. A. Wentworth. 421 pages; price $1.12. Ginn & Co.

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Mr. A. H. McConnell having charge of the designing, manufacture and testing of apparatus, graduated from Evanston, Illinois, High School, and completed his education at Northwestern University. After several years actual experience at the bench and in the testing laboratory under the supervision of Mr. Alfred L. Robbins, a pioneer apparatus designer and manufacturer and a recognized authority, he became one of the founders of the Central Scientific Company in 1900.

Mr. Herbert C. Arms, in charge of the domestic and foreign purchases, stock and shipping, is a graduate of the University of Illinois, class of 1895, having previously specialized in Physics and graduated with highest honors from the Hyde Park High School of Chicago, Ill. For years prior to graduation he spent part of his vacation in the shop and office of the W. A. Olmsted Scientific Company, long and favorably known among science teachers of this country, of which firm his father, Mr. Charles H. Arms, was a member. For five years prior to entering the Central Scientific Company, Mr. Arms held a similar position with the Chicago Laboratory Supply & Scale Company.

Mr. John M. Roberts, in charge of the Sales Department, after graduating from the University of Missouri, class of 1894, took two years of graduate work in Science in the University of Chicago; this was followed by six years experience as principal and science teacher in Missouri, after which he spent several years in the employ of the Chicago Laboratory Supply & Scale Company.

Mr. A. H. Standish, General Business Manager, after fifteen years experience with one of Chicago's largest wholesale houses as general attorney and assistant credit manager, purchased an interest in and for

three years was the successful business manager of the Chicago Laboratory Supply & Scale Company.

Mr. Herbert J. Fox, directly in charge of the Chemical Department, a registered Pharmacist in Illinois and Iowa, graduated from the University of Iowa School of Pharmacy in 1880, for four years had charge of the Chemical store room of the University of Chicago, where he made all the reagents used in the Chemical Laboratories and the stains, lacquers and cements used by the Department of Biology of that Institution.

Neither this sketch nor any History of the development of the Apparatus business in the West would be complete without mention of Mr. Frank Aronson, for the past two and a half years Superintendent and Foreman of the Factory of the Central Scientific Company. For more than twenty-five years continuously he has, either as superintendent for other manufacturers or for himself, been in this business. John A. Stenberg & Co., The National School Furnishing Co., W. A. Olmsted Scientific Co., Walmsley, Fuller & Co., and the Chicago Laboratory Supply & Scale Co., were all largely indebted to his experience, skill and inventive genius during the years of their existence and there are but few apparatus makers in the middle West that have not received part of their training from "Frank."

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This is exclusive of their extensive Laboratory Furniture establishment, where they manufacture a line of standard Chemical Desks, Physics Tables, Apparatus Cabinets, etc. This department of their business was the result of many enquiries from their customers for laboratory furniture of approved design, which trade has heretofore been handled by the building contractors. Their complete Furniture Catalogue, the first ever published, will be of interest to all science teachers and assist in solving one of the difficulties in equipping a new laboratory.

By experience and discrimination they have selected a number of the most reliable foreign manufacturers of scientific apparatus and chemical glassware through whom they are giving exceptionally quick service on Duty Free lists, especially Analytical Balances, Spectral Apparatus, etc., which are carried in stock for them in Europe. Their establishment is at all times open to inspection by Science Teachers.

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THE PUBLISHERS of SCHOOL SCIENCE AND MATHEMATICS purpose to issue the magazine on the first of each month from October to June, inclusive. Material to appear in any number must be in the Editor's hands not later than the *first of the second month* immediately preceding.

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It is hoped that our friends and readers will try to increase the circulation of School Science and Mathematics among their friends and acquaintances. Sample copies will gladly be mailed to addresses furnished.

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" " Vol. III.	2.00
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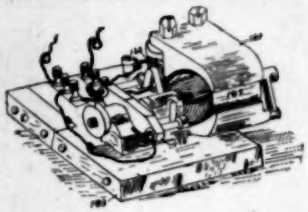

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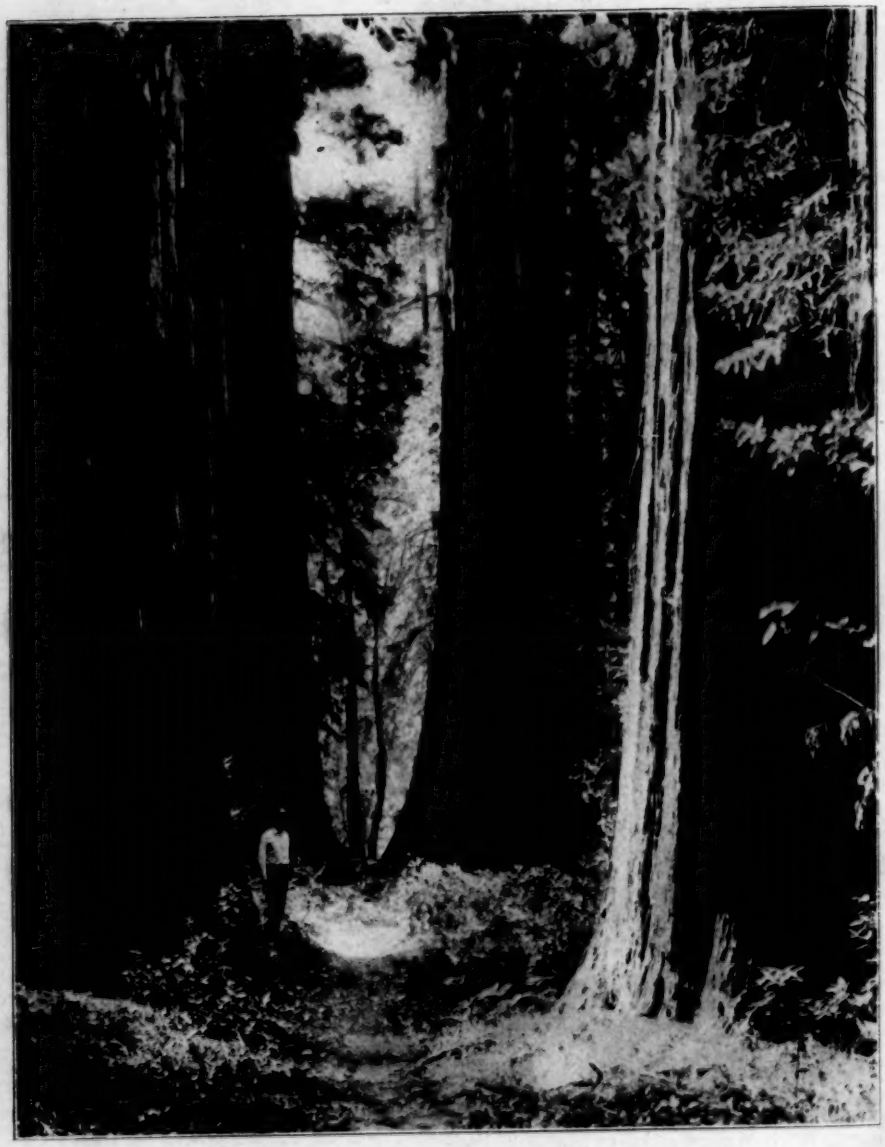

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